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Dynamica

Exam

This is an open book exam. You can use any book, any lecture note, and any notes from the werkcollege. But you may not consult anybody else, apart from the docenten (if, for example, you have problems with the language).
The exam consists of 7 problems. As you have 3.5 hours (09:00-12:30), this means 30 minutes per problem, so don't panic.
The total number of points in this tentamen is 100. You do not need to solve all problems correctly to get a cijfer 10.
IMPORTANT: Use symbolic quantities throughout your calculations; insert numerical values only at the very end. Not following this rule may lead to substraction of points.
Er bestaat van dit tentamen ook een nederlandse versie. De nederlandse en de engelse versie zijn identiek.

1) **Loop-the-loop (15 points)**

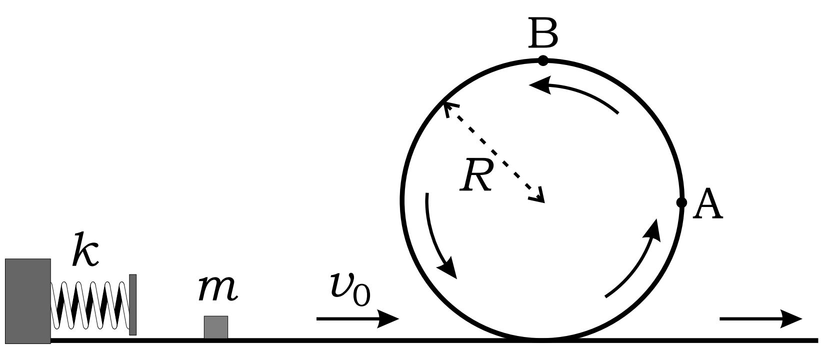
A sliding particle of mass m enters a circular loop-the-loop track of radius R with an initial velocity v_0 (see figure). There is no friction.

(a) Determine v_0 if the particle reaches point A, at a height R, and thereafter starts sliding down again.

(b) What is the minimal kinetic energy the particle must have to reach the highest point (B) of the track?

The particle is launched into the track by a spring of stiffness k.

(c) Determine the interval of spring compressions δ for which the particle will lose contact with the loop-the-loop track.



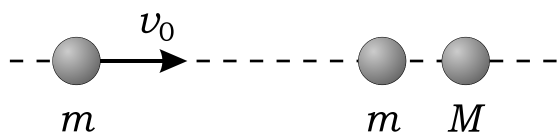
2) **Colliding spheres (15 points)**

The two spheres on the right of the figure are slightly separated and initially at rest; the left sphere is incident with speed v_0.
For (a) and (b) assume head-on elastic collisions.

(a) If $M \leq m$, show that there are two collisions and find all final velocities.

(b) If $M > m$, show that there are three collisions and find all final velocities.

(c) Find all final velocities if the balls collide completely inelastically and give an expression for the energy loss.



3) **Rolling cylinder (15 points)**

The weight of a steel cylinder of radius R is reduced considerably by drilling seven holes, each of radius a (see figure). One hole is located exactly in the center; the other six holes are distributed such that their centers coincide with the corners of a regular hexagon of edge length $R/2$. The length and mass of the cylinder are L and M respectively.

(a) Show that the rotational inertia of the cylinder about its major axis is given by

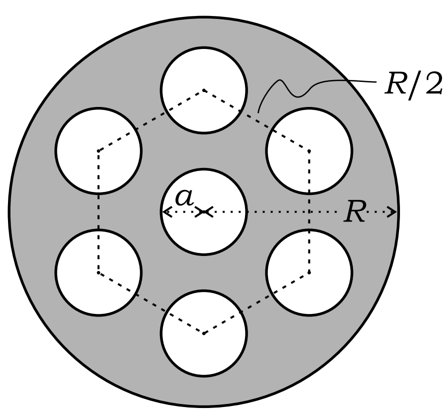
$$I = \frac{1}{2} M \frac{R^4 - 3R^2a^2 - 7a^4}{R^2 - 7a^2}. \tag{1}$$

The cylinder rolls, without slipping, from an inclined slope with an angle α with respect to the horizontal plane.

(b) Find the acceleration of the cylinder.

Let $R = 16.0$ cm; $a = 3.0$ cm, $L = 10$ cm, $\rho = 8.5 \cdot 10^3$ kg/m^3, and the angle $\alpha = 20$ degrees.

(c) What is the minimal value of the static coefficient of friction to prevent the cylinder from slipping?

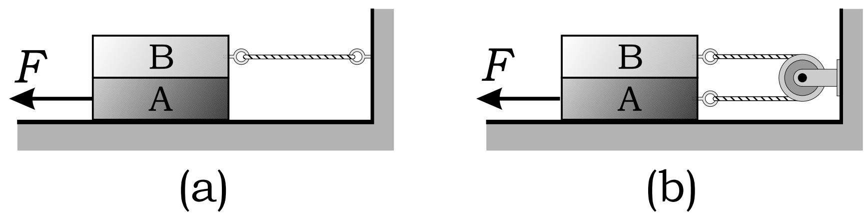


4) **Two blocks (10 points)**

Each of the two blocks in figures (a) and (b) has a mass M. The coefficient of kinetic friction at all surfaces of contact is μ. A horizontal force F is applied to the bottom block A such that it starts to move.

(a) Determine the acceleration of the bottom block in the situation depicted in figure a: a rope is connecting the top block to the wall.

(b) Determine the acceleration of the bottom block in the situation depicted in figure b: a rope connects A and B via a pulley which is mounted on the wall.



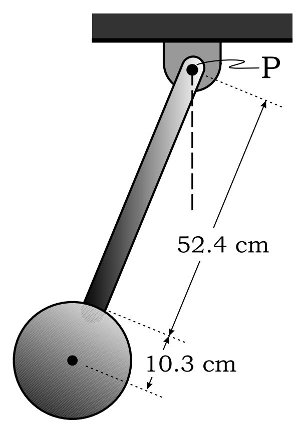
5) **Pendulum (15 points)**

A pendulum consists of a uniform disk of radius $r_d = 10.3$ cm and mass $m_d = 488$ g attached to a 52.4-cm-long uniform rod with mass $m_r = 272$ g (see figure).

(a) Determine the distance between the pivot P and the center-of-mass of the pendulum.

(b) Calculate the rotational inertia of the pendulum around the pivot.

(c) Calculate the small-angle period of oscillation.



6) **Rocket (10 points)**

A rocket has an initial mass of $m = 3.25 \cdot 10^4$ kg including its fuel (see figure). The fuel is burned at a constant rate and is expelled from the rocket at a relative speed of 900 m/s. The rocket is launched from rest into the vertical direction. Neglect the effects of air resistance and assume that g is constant.

(a) Derive an expression for the acceleration of the rocket as a function of time.

(b) Determine the (constant) rate at which the fuel has to be burned so that its thrust gives the rocket a speed of 60 m/s in 10 s starting from rest.

7) **Four masses (20 points)**

Four equal point masses (m each) are located at the corners of a square of edge length a, far away from any other masses (see left figure). The four masses rotate around their common center of mass in the same direction, such that they all follow the same circular orbit.

(a) What is the period of rotation in order to keep the orbit stable? (NB: the distances between the masses stay the same.)

An observer notes that the rotation speed of the masses is five times as large as the one calculated in (a) and attributes this to the presence of a spherical object of mass M and radius R_0 in the center of the square. Assume that the object is uniform.

(b) Determine M.

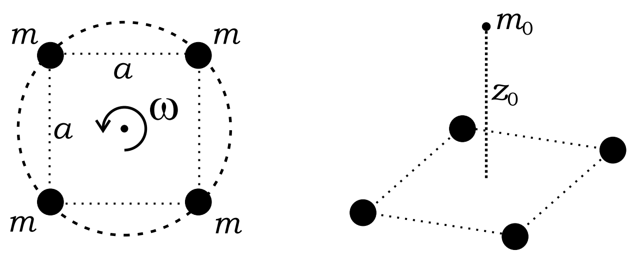
A point particle with mass m_0 on the symmetry axis is released from rest at a distance $z_0 (> R_0)$ from the center of mass of the square (see right figure).

(c) Show that the magnitude F of the force on the particle as a function of its position z along the axis is equal to

$$F = \frac{Gm_0M}{z^2} + \frac{4Gm_0mz}{(z^2 + a^2/2)^{3/2}}. \tag{2}$$

Here z is the distance between particle and the center of the square.

(d) Calculate the velocity with which the particle collides with the object in the center of the square.



Hint: in solving this problem the following integral may be useful:

$$\int \frac{x}{(x^2 + c^2)^q}\, dx = \frac{-1}{2(q-1)} \frac{1}{(x^2 + c^2)^{q-1}} \qquad \text{for all } q \neq 1 \tag{3}$$