Re-Examination: Mathematical Programming I (191580250)

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Ex. 1 Let $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times n}$ be symmetric matrices and let $\mathbf{Q} \in \mathbb{R}^{n \times n}$ be a non-singular matrix such that

$$\mathbf{Q}^T \mathbf{A} \mathbf{Q} = \mathbf{B}.$$

Show that $\mathbf{A}$ is positive semidefinite if and only $\mathbf{B}$ is positive semidefinite.

Ex.2 Consider the linear program in $\mathbb{R}^2$:

$$(P) \quad \max_{\mathbf{x} \in \mathbb{R}^2} \mathbf{c}^T \mathbf{x} \quad \text{s.t.} \quad \begin{aligned} -x_1 &\le 0 \\ -x_2 &\le 0 \\ x_1 + x_2 &\le 2 \end{aligned}$$

with $\mathbf{c} = (2, 1)^T$.

(a) Sketch the feasible set of P and show that the point $\overline{x} = (2, 0)^T$ is a solution of P.

(b) Give all solutions of P for the case that instead of $\mathbf{c} = (2, 1)^T$ we choose $\mathbf{c} = (1, 1)^T$.

Ex. 3 With $S^{n \times n} := \{\mathbf{A} \mid \mathbf{A} \in \mathbb{R}^{n \times n} \text{ is symmetric}\}$, consider the function $f : S^{n \times n} \to \mathbb{R}$, defined by:

$$f(\mathbf{A}) = \max_{\mathbf{x} \in \mathbb{R}^n, \mathbf{x}^T \mathbf{x} = 1} \mathbf{x}^T \mathbf{A} \mathbf{x}\,.$$

Show that $f(\mathbf{A})$ is a convex function.

Ex. 4 Let $f : \mathbb{R}^n \to \mathbb{R}$, be a C^1-function. Show that f is convex if and only if the following inequality holds:

$$\left(\nabla f(x) - \nabla f(x')\right)^T (x - x') \ge 0 \quad \text{for all } x, x' \in \mathbb{R}^n\,.$$

(Hint: For "$\Leftarrow$" use the mean-value relation:
$f(x) - f(x') = \nabla f(x' + \lambda(x - x'))^T (x - x')$ for some $\lambda \in (0, 1)$.
Also recall that $\nabla f(x)$ is a column vector.

Ex. 5 Given the function $f(\mathbf{x}) = \frac{1}{2}x_1^4 + 2x_1x_2 + 2x_1 + (1 + x_2)^2$

(a) Determine the critical points and the local minimizer(s) of f.

(b) Does there exist a global minimizer of f (on $\mathbb{R}^n$).

Ex. 6 We wish to find the minimizer of the quadratic function $q(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x}$ with - positive definite matrix **A**.

(a) Show that for any starting point $\mathbf{x}_0$ the Newton method finds the minimizer of q in one step.

(b) Let us apply the Quasi-Newton method. Suppose that this method produces the iterates x_k, the search directions $\mathbf{d}_k$ and the matrices $\mathbf{H}_k$, $k = 0, 1, \ldots$ Show that the relation holds:

$$\mathbf{H}_k^{-1} d_j = \mathbf{A} d_j \text{ , for all } j = 0, \ldots, k-1 \text{ ,}$$

and after n steps we have $\mathbf{H}_n = A^{-1}$.

Hint: Use the relation $\mathbf{H}_k\gamma_j = \delta_j$, $0 \le j \le k-1$ *where* $\gamma_j = \mathbf{g}_{j+1} - \mathbf{g}_j$, $\delta_j = \mathbf{x}_{j+1} - \mathbf{x}_j$.

Normering:

1	a	:	5	2	a	:	4	3	a	:	5	4	a	:	7	5	a	:	4	6	a	:	3
					b	:	2										b	:	2		b	:	4

Points: 36+4 =40

The scipt 'Mathematical Programming I'
may be used during the examination. Good luck!