Random Signals and Systems (157108)

Hajek's lecturenotes and additional course notes and your own notes & homework can be consulted (but other notes are not allowed)

Date: 31-08-2009
Place: Citadel 228 ...
Time: 9:00–12:00

1. Is the sample space Ω independent from itself?
2. Prove Hajek's Lemma 1.1.6(b), so for the case that $B_1 \supset B_2 \supset B_3 \cdots$.
3. Suppose X and Y are independent and let $f_X(x)$ and $f_Y(y)$ denote the respective probability density functions. Define $U = X + Y$, $V = 2Y$. Express the joint pdf $F_{UV}(u, v)$ in terms of f_X and f_Y.
4. Suppose $f_{XY}(x, y) = g(x - y)h(y)$ for certain functions g and h.
   (a) Show that $\int_{-\infty}^{\infty} g(x)dx \int_{-\infty}^{\infty} h(y)dy = 1$
   (b) Express $f_{X|Y}(x|y)$ in terms of g and h.
5. Is Brownian motion mean square differentiable?
6. Suppose Y_1, Y_2, Y_3 are uncorrelated and zero mean and that $\mathrm{E}\, Y_k^2 = k$. Suppose X is a stochastic variable such that $\mathrm{E}\, XY_i = 1$. Determine the linear MMSE $h(Y_1, Y_2, Y_3)$ of X.
7. Let $\{W_t\}_{t \geq 0}$ be Brownian motion with $\sigma = 1$, and take $X_t = \int_0^t W_\tau / \sqrt{\tau}\, \mathrm{d}\tau$ for $t > 0$.
   (a) Is $X_t := \int_0^t W_\tau / \sqrt{\tau} \mathrm{d}\tau$ well defined in mean square sense for $t \geq 0$?
   (b) Determine $R_X(s, t)$ for $s, t \geq 0$.
8. Consider the differential equation

   $$Y_t' + Y_t = U_t.$$

   Assume that U_t and Y_t and derivative Y_t' are m.s. continuous and jointly WSS.
   (a) Determine a relation between $R_{Y'Y}$, R_Y, R_{UY}. [Hint: mqultiply both sides of the differential equation with U_s and then take expectation of both sides.
   (b) Multiply both sides of the differential equation with Y_s and then take expectation of both sides.
   (c) Use the above to derive a differential equation that relates R_Y and R_U. [Hint: use that $R_{YY'} = -R_Y'$.]