Exam Game Theory (191521800)

University of Twente

Thursday November 10, 2016, 8:45-11:45h

This exam has 5 exercises.
Motivate all your answers! You may not use any electronic device.
This exam comes with a cheat sheet that contains most of the basic definitions. (See the last three pages.) Other necessary definitions are given in the questions. You are also allowed to bring your own cheat sheet (1 A4).

Noncooperative Game Theory

1. (12 points) Consider the (symmetric) bimatrix game given by $A = \begin{pmatrix} 0 & 15 \\ 8 & 13 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 10 \\ 17 & 15 \end{pmatrix}$.

   (a) Compute all Nash equilibria of this game.

   (b) Write down all conditions that define the correlated equilibria of this game, and exhibit a correlated equilibrium that is not a Nash equilibrium.

   (c) Explain why (in general bimatrix games) Nash equilibria are correlated equilibria.

   (d) Consider the bimatrix game given by $\bar{A} = \begin{pmatrix} 0 & 15 & 1 \\ 8 & 13 & 2 \\ 3 & 13 & 1 \end{pmatrix}$ and $\bar{B} = \begin{pmatrix} 2 & 10 & 5 \\ 17 & 15 & 15 \\ 2 & 1 & 30 \end{pmatrix}$.

   and show (by domination arguments) that the game $(\bar{A}, \bar{B})$ reduces to the game (A, B) from exercise (a) above.

   (e) Explain how correlated equilibria for the game $(\bar{A}, \bar{B})$ relate to correlated equilibria in the game (A, B). (Motivate your answer!)

Please turn over.

Cooperative Game Theory

2. (9 points) Consider the following three-person cooperative game (N, v).

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	3	4	5	9	10	8	15

   (a) Is this game superadditive? Is it convex?

   (b) Compute the core and the domination core. Are these equal?

   (c) Compute the Weber set. Which marginal vectors belong to the core?

   (d) Compute the nucleolus of this game.

3. (3 points) A cooperative game (N, v) is *additive* if $v(S \cup T) = v(S) + v(T)$ for all coalitions $S, T \subset N$ with $S \cap T = \emptyset$. Show that the Shapley value of an additive game (N, v) equals ϕ with $\phi_i = v(\{i\})$ for all $i \in N$.

Stochastic Game Theory

4. (6 points) Consider the following irreducible stochastic game with infinite horizon and average rewards:

state 1	-1 $\;(0,1)$	2 $\;(\frac{1}{2},\frac{1}{2})$
	1 $\;(\frac{1}{2},\frac{1}{2})$	-2 $\;(0,1)$

state 2	1 $\;(1,0)$

   (a) Player 1 uses the stationary strategy $\mathbf{f} = ((1,0),(1))$ and player 2 uses the stationary strategy $\mathbf{g} = ((0,1),(1))$. Show that the value vector equals $\mathbf{v}_\alpha(s, \mathbf{f}, \mathbf{g}) = 5/3$ for any s.

   (b) Take $\mathbf{w} = (2/9, -4/9)$ and $v = \mathbf{v}_\alpha(s, \mathbf{f}, \mathbf{g})$, $s \in S$, the average reward found in part (a). Show that the equation $\mathbf{w} + v\mathbf{1}_N = \mathbf{r}(\mathbf{f}, \mathbf{g}) + P(\mathbf{f}, \mathbf{g})\mathbf{w}$ holds. How do you interpret this equation?

5. (6 points)

   (a) Consider a zero-sum stochastic game. This game has a pair of strategies (π_*^1, π_*^2) satisfying

$$v_\beta(\pi^1, \pi_*^2) \leq v_\beta(\pi_*^1, \pi_*^2) \leq v_\beta(\pi_*^1, \pi^2)$$

   for all strategies π^1 and π^2. Show that this implies the existence of the value of the game.

(b) Let $\beta \in (0,1)$. Consider a transient stochastic game with $\sum_{s' \in S} p(s'|s, a^1, a^2) = \beta$ for all (s, a^1, a^2). Explain why such games may be interpreted as discounted stochastic games with discount factor β.

Total: 36 + 4 = 40 points