Solutions Exam 2017 (answers only)

$\boxed{1}$ a) $NE = \{((0,1),(1,0)),((1,0),(0,1)),((\frac{1}{5},\frac{4}{5}),(\frac{1}{5},\frac{4}{5}))\}$

b) $CE =$ all $p_{11}, p_{12}, p_{21}, p_{22} \geq 0$ w. $\sum p_{ij} = 1$ and

$p_{12} \geq 4p_{11}$, $p_{22} \leq 4p_{21}$, $p_{21} \geq 4p_{11}$, $p_{22} \leq 4p_{12}$,

For example $\begin{pmatrix} 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} \end{pmatrix}$. The latter has rank > 1, so it is not a NE.

c) none of the implications is true, for example take a game where player 1 has <u>one</u> strategy only, and players 2, 3 play a <u>prisoner's dilemma</u>. Then G and $\tilde{G}$ have <u>no</u> Nash equilibrium in common.

$$\begin{pmatrix} -1,-1 & -10,0 \\ 0,-10 & -9,-9 \end{pmatrix}$$

d) Same example as above, and note that the CE of a prisoner's dilemma is exactly (and only) the unique NE (show that!)

$\boxed{2}$ b) $C = \text{conv}\left\{\begin{pmatrix}-1\\-4\\-2\end{pmatrix}, \begin{pmatrix}-3\\-2\\-2\end{pmatrix}, \begin{pmatrix}-3\\-1\\-3\end{pmatrix}, \begin{pmatrix}-1\\-1\\-5\end{pmatrix}\right\}$

a) show that all m^σ, σ permutation, are $\in C$, then Weber Set $W \subseteq C$, so $W = C$, so game is convex.

d) $$y(N) = \sum_{i \in N} \frac{w(i)^2}{\sum_{j \in N} w(j)^2} w(N) = w(N).$$

i,j symm. $\Leftrightarrow$ $w(S \cup i) = w(S \cup j)$ $\forall S \not\ni i,j$. Now $w(\emptyset \cup i) = w(i)$, so $w(S \cup i) = w(S \cup j) \Rightarrow w(i) = w(j) \Rightarrow y_i(w) = y_j(w)$.

e) $y_1(v) = -7\left(\frac{9}{50}\right)$, $y_2(v) = -7\left(\frac{16}{50}\right)$, $y_3(v) = -7\left(\frac{25}{50}\right)$

Now
$$\left.\begin{array}{l} -3 \leq y_1(v) \leq -1 \\ -4 \leq y_2(v) \leq -1 \\ -5 \leq y_3(v) \leq -2 \end{array}\right\} \Rightarrow y(v) \in C.$$

3. Use the following:

* $i \notin T \Rightarrow i$ is null player $\Rightarrow \phi_i(u_T) = 0$
* $i, j \in T \Rightarrow i, j$ are symmetric $\Rightarrow \phi_i(u_T) = 1/|T|$.

4. (a) MR-decision rule: Markovian & randomized

e.g. $d(1) = \begin{cases} T & \text{with prob. } \frac{1}{2} \\ B & \text{with prob. } \frac{1}{2} \end{cases}$ (player 1)

$d(2) = a$

HD-decision rule: history-dependent & deterministic

e.g. $d(1) = \begin{cases} T & \text{if } B \text{ was selected previously in } s=1 \\ B & \text{else} \end{cases}$

$d(2) = a$.

(b) $v_\beta = (2\frac{1}{2}, 2)$

Optimal strategies:

player 1 (f^∞) with $f = ((\frac{1}{4}, \frac{3}{4}), (1))$

player 2 (g^∞) with $g = ((\frac{1}{2}, \frac{1}{2}), (1))$.

(c) Use the following:

Horizon $T=1$: $\quad v_{\beta,1}(s) = \max_{a \in A_s} \{ r(s,a) + \beta \sum_{s'} p(s'|s,a) v_\beta(s) \}$

Infinite horizon:

$$v_\beta(s) = \max_{a \in A_s} \{ r(s,a) + \beta \sum_{s'} p(s'|s,a) v_\beta(s) \}.$$

5. States $s = (j, a, b, t)$

j = player who has the dice $(j = 1, 2)$

a = accumulated score of player 1, $a < 100$

b = acc. score of player 2, $b < 100$

t = turn score of player j. $\begin{cases} \text{if } j=1: & 0 \leq t \leq 105-a \\ \text{if } j=2: & 0 \leq t \leq 105-b \end{cases}$

(accumulated score of a player ≥ 100, then game over)

| Actions if at play: to roll, or to stop

if not at play: to wait

| Payoff: goal: maximize probability of winning ← or different

(zero-sum) $r_1(s) = \begin{cases} 1 & \text{if } s = (1, a, b, t) \text{ with } a+t \geq 100 \\ 0 & \text{else} \end{cases}$

$r_2(s) = \begin{cases} 1 & \text{if } s = (2, a, b, t) \text{ with } b+t \geq 100 \\ 0 & \text{else} \end{cases}$

| Prob. transitions

Player 1. action = to stop

$$(1, a, b, t) \xrightarrow{\text{prob. } 1} (2, a+t, b, 0).$$

action = to roll

$$(1, a, b, t) \xrightarrow{1/6} (2, a, b, 0) \quad \text{(ace)}$$

$$(1, a, b, t) \xrightarrow{1/6} (1, a, b, t+i), \quad i = 2, \dots, 6.$$

Similarly for player 2.