Exam Game Theory (191521800)
University of Twente
January 27, 2022

This exam has 8 exercises.
Motivate all your answers! **You may not use any electronic device.**
This exam comes with a cheat sheet that contains most of the basic definitions. (See the last pages.) Other necessary definitions are given in the questions. You are also allowed to bring your own cheat sheet (1 A4, one-sided).

Noncooperative Game Theory

1. (2+2 points) Consider the bimatrix game given by

$$(A, B) = \begin{pmatrix} 7,16 & 8,15 \\ -1,1 & 9,8 \end{pmatrix}$$

   (a) Compute all Nash equilibria of this game.

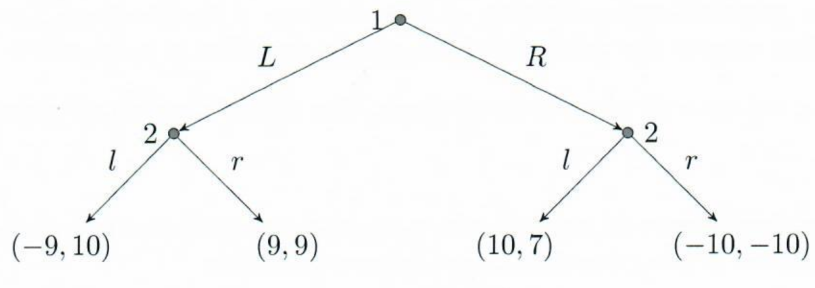
   (b) Write down all conditions that define the correlated equilibria of this game.

2. (3 points) Consider a game G with three players $1, 2, 3$ with corresponding strategy sets S_1, S_2, S_3, and payoff functions u_1, u_2, u_3, respectively.

   Let G' denote the game where player 2 and 3 are combined. So, G' is a two player game with strategy sets S_1 and $S_2 \times S_3$, respectively. The payoff functions are defined in the obvious way: The payoff for player 1 is as in the original game, the payoff of the combined player is the sum of the payoff of player 2 and 3 in the original game.

   What can you say about the relation between Nash equilibria in G and G'? In particular, assume $(\sigma_1, \sigma_2, \sigma_3)$ is a Nash equilibrium in G, can you conclude that $(\sigma_1, (\sigma_2, \sigma_3))$ is a Nash equilibrium in G'? And what about the converse?

3. (2+2+1 points) Consider the following extensive form game (with perfect information and perfect recall).



   (a) Give the strategic form (bimatrix) representation of this extensive form game.

   (b) Compute all subgame perfect equilibria.

   (c) Give a Nash equilibrium that is not a subgame perfect equilibrium

Cooperative Game Theory

4. (2+3+1 points) Consider the following three player cooperative game (N, v).

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	2	5	4	15	18	14	24

   (a) Is the game a convex game? Is it super-additive? Give proofs.

   (b) Compute and depict the core $C(N, v)$, as well as the domination core $DC(N, v)$ of that game. What are the extreme points (vertices) of the core?

   (c) What is the maximal value $v(\{2, 3\})$ such that $C(N, v) \neq \emptyset$?

5. (4+2 points) Consider the following solution value ψ defined for a cooperative game (N, v) with $|N| = n$. For all $i \in N$, let i's payoff be the average of all marginal contributions of i,

$$\psi_i(N, v) := \frac{1}{2^{n-1}} \sum_{S: i \notin S} \left(v(S \cup \{i\}) - v(S) \right).$$

   (a) Show that ψ is symmetric, additive and fulfils the null player property, but may fail to be efficient (in general).

   (b) Show that ψ equals the Shapley value when $n = 2$.

Stochastic Game Theory

6. (4+2 points) Consider the stochastic game situation below in which the players optimize their discounted rewards with discount factor $\beta = 4/5$.

1 $(1,0)$	0 $(1,0)$
0 $(0,1)$	2 $(0,1)$

$s = 1$

-2 $(0,1)$

$s = 2$

   (a) What is the value vector of this game and which strategies are optimal for the players?

   (b) Give the graphical representation of this game as a transient stochastic game with stopping probability; what is the value of the stopping probability in each state?

7. (2 points) Give an example of a stochastic game that is also a repeated game.

8. (2+2 points)

   (a) There are three classes of strategies in stochastic games. Select two of these classes and compare them: mention at least one difference and one similarity.

   (b) Consider an irreducible stochastic game with the average-rewards criterion. Player 1 uses the stationary strategy $(\mathbf{f}^\infty)$ and player 2 uses $(\mathbf{g}^\infty)$. A student has to find the average reward vector corresponding to these strategies and reports:

   "I find two solutions to the equation $\mathbf{w} + v\mathbf{1}_N = \mathbf{r}(\mathbf{f}, \mathbf{g}) + P(\mathbf{f}, \mathbf{g})\mathbf{w}$, namely the solutions $(\tilde{v}, \tilde{\mathbf{w}})$ and $(\breve{v}, \breve{\mathbf{w}})$ with $\tilde{\mathbf{w}} \neq \breve{\mathbf{w}}$. Because of this, I conclude that the average reward cannot be found."

   What is wrong in this student's answer?

Total: 36 + 4 = 40 points