Exam Queueing Theory — April 7, 2020 (13.45 – 16.45)

This is an open book exam. This means that you are allowed to use

- reader Queueing Systems
- reader Markovian Queueing Networks
- the slides of the lectures

In addition, you are allowed to use an ordinary (scientific) calculator, but a programmable or graphic calculator ('GR') is not allowed.
You are **not** allowed to use the Internet or any other reference during the exam.
At the end of the time reserved for the exam, or earlier if you decide to stop earlier, you must upload a scan of photos of your results via Canvas as a single pdf file and also send this file via email to r.j.boucherie@utwente.nl.
Clearly, the pdf file should contain scan or photos should of good quality, so that we can read your results. Scans or photos of insufficient quality will not be marked.
Following the test, there will be an oral exam for a randomly selected group of participants. This oral exam will last for at most 15 minutes per selected participant. If you are selected, you must participate in this oral exam. These oral exams will start Tuesday April 8 at 17:00. You must be available for these exams in the time slot 17:00 - 20:00. You will receive an email just before the start of your oral exam. The oral exam will be via Canvas conferences.
Please read the following paragraph carefully, and copy the text below it verbatim to your answer sheet.
By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.
Text to be copied and signed:
I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.

This exam consists of five problems. Please put your name and student number on each sheet of paper. Using a simple (not graphic) scientific calculator is allowed. Motivate your answers.

1. Consider the $M/G/1$-LIFO-PR queue. Let the Poisson arrival process have rate λ.

   In part (a) and (b), we assume that the service time distribution is a mixed Erlang distribution that is with probability $p_k = \frac{\mu^k}{k!}\mathrm{e}^{-\mu}$ the sum of k exponentials with rate ν, $k = 1, 2, \ldots$.

   a. Give the traffic intensity ρ, and give two different interpretations of this quantity.

   b. Give an explicit expression for the LST of the service time distribution.

   We will now assume that $\rho < 1$. We will now consider the general case for the service time distribution (that is in part (c) and (d) we will no longer assume that service times have a mixed Erlang distribution).

   c. Let $\widetilde{S}(s)$ denote the LST of the sojourn time. Give an expression for the LST of the sojourn time. Motivate your answer.

   d. Use the result from (c) to show that the mean number of customers in the $M/G/1$-LIFO-PR queue depends on the service time distribution only through the mean service time (and not the higher moments of the service time distribution). Motivate your answer.

2. In a workshop, a machine paints parts upon request from customers. The interarrival times between requests are i.i.d. negative exponential with rate λ. Requests are composed of one or two parts. With probability p_i a request is composed of i parts, $i = 1, 2$. Requests from the customers are handled in order of arrival. Painting of a part takes a negative-exponential time with rate ν. Upon completion of painting of a part, the part is immediately delivered to the customer.

   a. Give the stability condition for the workshop. Motivate your answer.

   b. Derive the distribution of the number of parts present in the workshop. Motivate your answer.

   c. Give the LST of the sojourn time until completion of a request of a customer (that is the time until completion of all requests from the customer). Motivate your answer.

3. Consider the $M/G/1$-FIFO queue. Give an expression for the LST of the length of a busy period when there is a fixed setup time T (i.e., when a customer arrives to an empty system, it takes a time T before the first service starts). Motivate your answer.

4. In a small and remote town an Internet cafe is established. About 4 customers per hour are expected to arrive according to a Poisson process and the duration of a session is exponentially distributed with mean 30 minutes. Assuming that customers leave immediately when they find all connections busy, are four connections sufficient to ensure that at least 90% of the customers can use the internet? Motivate your answer.

5. Consider a tandem network of 3 single-server FIFO queues. Jobs of type t, $t = 1, \dots, T$, arrive at queue 1 according to a Poisson process with rate λ. The service times of jobs in queue i are independent and identically distributed and have exponential distributions with rates μ_i, $i = 1, 2, 3$. Jobs that are served in queue 1 route to queue 2, jobs that are served in queue 2 route to queue 3, and jobs served in queue 3 leave the network. Queue 2 has a finite waiting room such that at most c_2 customers may be present simultaneously at queue 2 (waiting and in service). Queue 1 and queue 3 have unlimited waiting room.

   a. Assume that the network operates under the stop-protocol, see Example 3.4.6. Under this protocol, if queue 2 is saturated, i.e., in state $n_2 = c_2$, then the service at queue 1 and queue 3 is stopped and the arrival process to the network is stopped.

      Model this system as a network of queues and give the state space and the transition rates of the Markov chain that records the number and type of customers in the queues. Give the stability condition for this network.

   b. Assume that the network operates under the stop-protocol. Is queue 1 quasi-reversible? Is queue 2 quasi-reversible? Is queue 3 quasi-reversible? Motivate your answers.

   c. Now assume that the network operates under the push-out-protocol. Under this protocol, if a customer arrives to queue 2 in state $n_2 = c_2$, then the service of the customer in service is stopped and this customer in service routes to queue 3. The arriving customer at queue 2 joins the tail of the queue.

      Model this system as a network of queues and give the state space and the transition rates of the Markov chain that records the number and type of customers in the queues. Give the stability condition for this network.

   d. Assume that the network operates under the push-out-protocol. Is queue 1 quasi-reversible? Is queue 2 quasi-reversible? Is queue 3 quasi-reversible? Motivate your answers.

Norm: Exam grade = total/4

1				2			3	4	5				total
a	b	c	d	a	b	c			a	b	c	d	
2	2	3	2	2	3	3	4	3	3	3	3	3	+ 4 = 40