Exam Queueing Theory
Monday, May 17, 2010, 13.00–16.00.

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1. Jobs arrive at two parallel machines, labeled 1 and 2, according to a Poisson stream with a rate of 10 jobs per hour. The processing time is exponential with a mean of 4 minutes on machine 1 and with a mean of 8 minutes on machine 2. On arrival a job is send to machine 1 or 2 with equal probability. Each machine is processing jobs in order of arrival.

   a) Determine the mean production lead time (waiting time plus processing time) of a job on machine 1, a job on machine 2 and of an arbitrary job.

   b) Determine the long-run fraction of jobs with a production lead time greater than 30 minutes.

   Suppose that on arrival a job is send with probability p to machine 1 and with probability $1 - p$ to machine 2.

   c) Determine the value of p minimizing the mean production lead time of an arbitrary job.

2. A book store is selling the new comic "LOGICOMIX: An Epic Search for Truth." It keeps 2 copies of that book on stock. Customers who want to buy this book arrive according to a Poisson process with a mean of 1 customer per day. When a copy of LOGICOMIX is sold, the book store immediately places an order for a new copy at the supplier. The lead time is exactly 2 days. Customer demand is lost if there is no copy of LOGICOMIX on the shelf; the (disappointed) customer will then go to another book store.

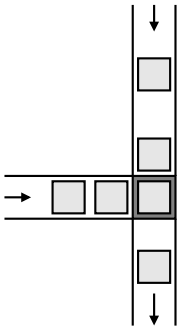
   a) Determine the long-run fraction of time there are 0, 1, or 2 copies of LOGICOMIX on the shelf.

   b) Determine the long-run fraction of customer demand that is lost.

   c) What is the minimal number of copies that should be kept on stock by the book store such that less than 10% of customer demand will be lost?

3. In a distribution center pallets with products are transported on an automatic conveyor system. In Figure 1 a junction is shown, where pallets from the West and North join the main conveyor belt. The time to pass the transfer point (dark square in Figure 1) is exactly 8 seconds for a pallet coming from the North. Pallets from the West first need to be lifted a little bit, and therefore the time to pass the transfer point is longer, i.e., it is exactly 12 seconds for pallets from the West. Pallets arrive at the transfer point according to a Poisson process, with a rate of 3 pallets per minute from the North and 2 pallets per minute from the West.

   a) Calculate the mean time to pass the transfer point (waiting time plus transfer time) for a pallet from the North and for a pallet from the West in case pallets are transferred in order of arrival.



Figuur 1: Junction of an automatic conveyor system transporting pallets.

b) Calculate the mean time to pass the transfer point for a pallet from the North and from the West in case pallets from the North are handled with non-preemptive priority over pallets from the West.

Now assume there are always pallets from the West waiting to be transferred.

c) Calculate the mean time to pass the transfer point for a pallet from the North in case these pallets are handled with non-preemptive priority over pallets from the West.

d) At what rate do pallets from the West pass the transfer point?

4. Customer orders for the production of 1 product arrive at a machine according to a Poisson process with a rate of 2 orders per day (8 hours). The production of a product consists of two phases: The first phase is standard and the same for all customers. The second phase, however, depends on the specific requirements of the customer. The first phase takes a constant time of 2 hours; the second phase is exponential with a mean of 1 hour. Orders are processed in order of arrival.

a) Determine the Laplace-Stieltjes transform of the processing time in hours.

b) Show that the Laplace-Stieltjes transform of the *residual* processing time in hours is given by

$$\widetilde{R}(s) = \frac{1 + s - e^{-2s}}{3s(1+s)} \,.$$

c) Show that the Laplace-Stieltjes transform of the waiting time in hours is given by

$$\widetilde{W}(s) = \frac{s(1+s)}{(4s-1)(1+s) + e^{-2s}} \,.$$

d) Determine the mean production lead time (waiting time plus processing time).

To reduce the production lead time the machine starts already with the production of the first phase when the system is empty. If upon completion of the first phase no order has arrived yet, the machine stops. When the first order arrives the machine can directly start with the customer specific second phase.

e) Determine the reduction in the mean production lead time.

Credits:

1a	b	c	2a	b	c	3a	b	c	d	4a	b	c	d	e
3	3	5	3	3	5	3	3	3	3	3	3	3	3	4