

Exam Queueing Theory
Monday, June 21, 2010, 13.00–16.00.

1. An insurance company has a call center, consisting of four operators, handling questions of customers. Calls of customers arrive according to a Poisson process with a rate of 40 calls per hour. The time needed to help a customer is exponentially distributed with mean 3 minutes.
 - a) Determine the limiting distribution of the number of customers in the system.
 - b) Determine the mean sojourn time of a customer.
 - c) What is the probability that a customer has to wait more than 2 minutes before receiving service by an operator?

A period during which, uninterruptedly, all operators are occupied is called a crowded period. A period during which, uninterruptedly, at least one operator is idle is called a quiet period.

- d) How long lasts on average a crowded period? And how long lasts on average a quiet period?
2. At the shop of a handyman, a person skilled at a wide range of repairs, customers arrive according to a Poisson process with a rate of 3 customers per hour. Two third of the customers have only one job for the handyman, while the remaining one third of the customers have two jobs for him. The service time of a job is exponentially distributed with a mean of six minutes. Customers are served in order of arrival. The sojourn time of a customer is defined as the time from the arrival of the customer at the shop of the handyman until the moment that all jobs of the customer have been completed. The waiting time of a customer is defined as the time from the arrival of the customer until the moment the handyman starts with the (first) job of the customer.
 - a) Determine the mean waiting time and the mean sojourn time of an arbitrary customer.
 - b) Determine the mean number of unfinished jobs in the shop of the handyman.

The handyman decides to give customers with only one job priority over customers with two jobs. However, if the handyman already works on a job of a customer with two jobs, he will finish all jobs of this customer before starting to work on a job of a new customer.

- c) Determine the mean waiting time of a customer with one job and the mean waiting time of a customer with two jobs.
 - d) Determine the mean number of unfinished jobs in the shop of the handyman.
3. A book store is selling the popular book “The Last Lecture” of Randy Pausch. It keeps 3 copies of that book on stock. Customers who want to buy this book arrive according to a Poisson process with a mean of 1 customer per day. When a copy of “The Last Lecture” is sold, the book store immediately places an order for a new copy at the supplier. Also

if there is no copy on the shelf, an order for a new copy will be immediately placed and the customer will wait for the first copy to become available from the supplier. The mean lead time at the supplier is 2 days.

- a) Determine the long-run fraction of time there are i outstanding orders.
 - b) Determine the mean number of books on the shelf.
 - c) What is mean waiting time of a customer?
4. A production line consists of two machines in series. Machine 1 works uninterruptedly, there is always supply of new products. The production time of a product on machine 1 is with probability $2/3$ exponentially distributed with a mean of 1 minute and with probability $1/3$ exponentially distributed with a mean of 3 minutes. The production time of a product on machine 2 is exponentially distributed with a mean of 1 minute. In between machine 1 and machine 2 there is an infinite buffer. As soon as machine 1 has finished a product, this product is placed into the buffer. As soon as machine 2 has finished a product, it takes the next product out of the buffer (if available). If at that moment the buffer is empty, machine 2 waits until machine 1 has finished the next product.
- a) Determine the fraction of time machine 2 is working.
 - b) Determine the distribution of the sojourn time of a product at the second stage of the production line (i.e., the time period starting at the moment that a product enters the buffer and ending at the moment that the production time of the product at the second machine is completed).
 - c) Determine the mean number of products in the buffer at an instant a product is placed into the buffer.
 - d) Determine the mean number of products in the buffer at an arbitrary instant.
5. Customers arrive at a server according to a Poisson process with a rate of 6 customers per hour. As soon as there are no customers the server leaves to do something else. He returns when there are 2 customers waiting for service again. It takes exactly 5 minutes for him to return. The service time of a customer is uniformly distributed between 5 and 10 minutes.
- a) Determine the mean sojourn time of a customer.
 - b) Determine the mean duration of a busy period, i.e., a period during which the server is servicing customers without interruptions.

Credits:

1a	b	c	d	2a	b	c	d	3a	b	c	4a	b	c	d	5a	b
2	2	2	3	3	2	2	2	2	2	3	2	3	2	2	3	3