Exam for LNMB/Mastermath Course on Scheduling
26 May 2014

This exam consists of:

- **2 pages.**
- **5 questions.**
- You can obtain a total of 50 points. Your exam grade will be the points you obtained divided by 5.

When a proof is asked, please provide a mathematically sound proof, short but precise. Unless stated otherwise, you are always expected to (briefly) explain your answer.

In case the objective function is not explicitely specified, it is supposed to be a regular objective function, unless stated otherwise.

Exercise 1 (10 points)
Consider the single machine scheduling problem with sequence dependent setup times. In this problem n jobs need to be processed on a single machine. When job k is processed directly after job j, a setup of the machine is needed that takes s_{jk} time units. Moreover, the time it takes to process job j is p_j. The goal is to minimize the makespan. In the $\alpha|\beta|\gamma$-notation, this problem is denoted by $1 \mid s_{jk} \mid C_{\max}$.

Show that this problem is NP-hard by a reduction from the traveling salesman problem (TSP).

Exercise 2 (10 points)
Consider the problem $1 \mid \mid \sum C_j^2$.

Determine an algorithm that solves this problem to optimality and show the correctness of your algorithm.

Exercise 3 (10 points)
Consider the problem O$||C_{\max}$ and the following list-scheduling algorithm. Whenever a machine becomes idle, it starts processing an available operation that has not been processed on this machine. Note that an operation is available when its job is not being processed on another machine. Ties are broken according to the jobs' positions in a predefined list.

Show that this list-scheduling algorithm has a performance guarantee of 2.

Exercise 4 (10 points)
Consider the problem $1 \mid d_j = d \mid \sum_j (E_j + T_j)$, where the earliness of a job is defined as $E_j = \max\{d_j - C_j, 0\}$ and the tardiness of a job is defined as $T_j = \max\{C_j - d_j, 0\}$, i.e., $E_j + T_j = |C_j - d_j|$.

Show that in an optimal schedule there is no unforced idleness between two consecutive jobs.

Exercise 5 (10 points)
Consider the stochastic scheduling problem $\text{P} \mid \mid \mathbb{E}\left[\sum w_j C_j\right]$ with four jobs. The processing times of the jobs are independently distributed and have as possible realizations 1 or 19001. The weights and processing time distributions are given in the following table.

job	w_j	$x = 1$	$x = 19001$
job 1	1	Pr $[P_1 = x] = 1$	Pr $[P_1 = x] = 0$
job 2	1	Pr $[P_2 = x] = 1$	Pr $[P_2 = x] = 0$
job 3	10	Pr $[P_3 = x] = .999$	Pr $[P_3 = x] = .001$
job 4	10	Pr $[P_4 = x] = .999$	Pr $[P_4 = x] = .001$

(a) Determine an optimal policy for the case of $m = 1$.

(b) Determine an optimal policy for the case of $m = 2$.

(c) Suppose that there is now also a fifth job: $w_5 = 4000$ and Pr $\left[P_5 = 10^{-5}\right] =$ Pr $\left[P_5 = 10^{5}\right] = 0.5$.
Determine an optimal policy for $m = 2$ that needs to schedule all 5 jobs.