Course 191551200 "Scientific Computing"
test T_1

February 27, 2013, 14:45–14:05

Your name: _____

Your student number: _____

Space for your drafts (will not be checked)

1. (20 points[1]) Give the definition of the LU factorization with partial pivoting, indicating for which matrices it can be computed. Give a 3×3 matrix for which the Gaussian elimination process with partial pivoting will break down (i.e., terminate prematurely without succes).

2. (25 p) After carrying out three steps of the Gaussian elimination process with partial pivoting of a matrix $A \in \mathbb{R}^{3\times3}$, the following matrices are obtained: $P_1 = P_2 = I$,

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ -131 & 1 & 0 \\ 141 & 0 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -232 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 311 & 312 & -313 \\ 0 & -322 & 323 \\ 0 & 0 & 333 \end{bmatrix}.$$

Write down (without computing them) matrices L and P such that $PA = LU$.

3. (10 p) Give a definition of the SVD, indicating for which matrices it exists.

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[1]Total number of points is 100.

Space for your drafts (will not be checked)

4. (35 p) The Schur decomposition of a 3×3 matrix A is given by the matrices

$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}, \quad T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Give the eigenvalues and corresponding eigenvectors of A. After that determine the SVD of A.

5. (10 p) Provide the QR factorization of $A = \begin{bmatrix} 0 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$.