Course 19.155120.0 "Scientific Computing" test T_3

June 10, 2014, 15:45–16:15

Your name: __________________________________

Your student number: __________________________________

Space for your drafts (will not be checked)

Q1 Consider, for $F : \mathbb{R}^n \to \mathbb{R}^n$, the following problem: find $x \in \mathbb{R}^n$ such that

$$F(x) = 0. \tag{1}$$

(5 p) (a) Formulate, without proof, the fixed point iteration for this problem if the fixed point mapping is $K(x) = x - F(x)$.

(15 p) (b) Formulate, without proof, a condition which is sufficient for the fixed point iteration to converge locally.

(20 p) (c) Consider the problem (1)

$$F(x) = \frac{1}{3} x * x - \frac{3}{2} x, \tag{2}$$

where $*$ is the elementwise multiplication of the two vectors. For example if $x = (1, 2, 3)^T$ and $y = (2, 2, 1)^T$ then $x * y = (2, 4, 3)^T$. Consider the following two fixed point iterations:

$$\text{iteration I: } K(x) = x - F(x), \qquad \text{iteration II: } K(x) = x + F(x).$$

Which iteration would you prefer to find solutions x of problem (1),(2) such that $x \approx 0$? Of course, $x = 0$ is a solution but the iterative scheme does not know this.

(10 p) (d) [Make this exercise as last.] Propose a preconditioner matrix $M \in \mathbb{R}^{n \times n}$ such that the preconditioned nonlinear problem (1),(2) could be solved with both fixed point iterations given above.

Space for your drafts (will not be checked)

(20 p) Q2 Consider matrix

$$M = \begin{bmatrix} A & A & A \\ B & B & B \\ A & A & A \end{bmatrix} \in \mathbb{R}^{300\times 300},$$

where the matrices $A, B \in \mathbb{R}^{100\times 100}$ are given. Write down a formula which defines M with the help of the Kronecker product in terms of A, B and some 3×3 matrices.

(10 p) Q3 (a) For sufficiently smooth $F : \mathbb{R}^n \to \mathbb{R}^n$ write down the first two terms of the Taylor series of $F(x + \delta w)$, $x, w \in \mathbb{R}^n$, $\delta \in \mathbb{R}$, around point x.

(10 p) (b) Using the Taylor expansion write down an approximation for the Jacobian matrix-vector product $F'(x)w$.