Time Series Analysis (& SI)—191571090

Date:	24-07-2020
Place:	At home!
Time:	08:45–11:45 (till 12:30 for students with special rights)
Course coordinator:	G. Meinsma
Allowed aids during test:	see below

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer! You are not allowed to use a calculator.

Integrity statement Please read the following paragraph carefully.

By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

The only allowed sources for this test are:

- the lecture notes "Time Series Analysis and System Identification" (pdf or printed)
- the slides (pdf or printed)
- electronic devices, but only to be used:
  - for downloading the test and afterwards uploading your work to Canvas
  - to show the test/book/slides on your screen
  - to write the test (in case you prefer to use a tablet instead of paper to write on)

problem:	1	2	3	4	5	6
points:	4	2+2+2+2+2	2+2+3+2+3	3	3	4

Exam grade is $1 + 9p/p_{\max}$.

P.T.O.

A. **Copy the following text verbatim to the first page of your work (handwritten) and sign it. If you fail to do so, your test will not be graded:**

> *I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.*

B. **What programme do you follow (AM, AM+AP, AM+TCS, Minor, ...)**

C. **Are you entitled to extra time? (We will check this with CES.)**

---

1. Suppose the second-order MA scheme $X_t = b_0\epsilon_t + b_1\epsilon_{t-1} + b_2\epsilon_{t-2}$ has covariance function

$$\gamma(0) = 17, \qquad \gamma(1) = 0, \qquad \gamma(2) = -4.$$

Suppose $\sigma_\epsilon = 1$. Determine all possible b_0, b_1, b_2.

2. Let $b \in \mathbb{R}$. Suppose ϵ_t is a white noise process with mean zero and variance σ_ϵ^2. Let

$$(2 - \mathrm{q}^{-1} + \mathrm{q}^{-2})X_t = (b + \mathrm{q}^{-1})\epsilon_t.$$

(a) For which b is X_t is asymptotically wide-sense stationary.

(b) For which b is this scheme invertible?

(c) Determine the one-step ahead predictor of X_t (you may assume that b is such that the scheme is stable and invertible.)

(d) Determine the two-step ahead predictor of X_t (you may assume that b is such that the scheme is stable and invertible.)

(e) Determine the spectral density of $X_t - \hat{X}_{t|t-1}$.

3. Suppose $X_1, \dots, X_N$ are mutually independent stochastic variables with the same probability density function

$$f(x) = \begin{cases} \frac{4x}{\lambda^2}\,\mathrm{e}^{-2x/\lambda} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

for some parameter $\lambda > 0$. It may be shown that $\mathbb{E}\,X_t = \lambda$ and $\mathbb{E}\,X_t^2 = \frac{3}{2}\lambda^2$.

   (a) Show that $\operatorname{var}(X_t) = \lambda^2/2$.

   (b) Give the joint probability distribution of $X_1, \dots, X_N$.

   (c) Determine the maximum likelihood estimator $\hat{\lambda}$ of λ given $X_1, \dots, X_N$.

   (d) Is this estimator $\hat{\lambda}$ unbiased?

   (e) Is this estimator $\hat{\lambda}$ efficient?

4. The course does not cover parametric estimation of spectral densities, but this can be done: propose a method using AR-models to estimate spectral densities based on (as usual) N samples $X_0, \dots, X_{N-1}$ of a wide sense stationary process X_t.

5. **System identification:** Is it wise to use $u_t = (-1)^t$ as input for system identification of an LTI system such as shown in Fig. 6.1 of the lecture notes?

6. **System identification:** Consider the standard system identification problem (e.g. $Y_t = \sum_m h_m U_{t-m} + V_t$) and assume that

   - $U_t = (1 + \frac{1}{2}\,\mathrm{q}^{-1})\epsilon_t$ in which ϵ_t is some zero mean white noise process,
   - V_t is zero mean white noise,
   - U_t, V_t are uncorrelated stochastic processes,
   - Extensive testing says that $\gamma_{YU}(0) = 1$ and that $\gamma_{YU}(k) = 0$ for all $k \neq 0$.

   Determine the impulse response h_t of the system.