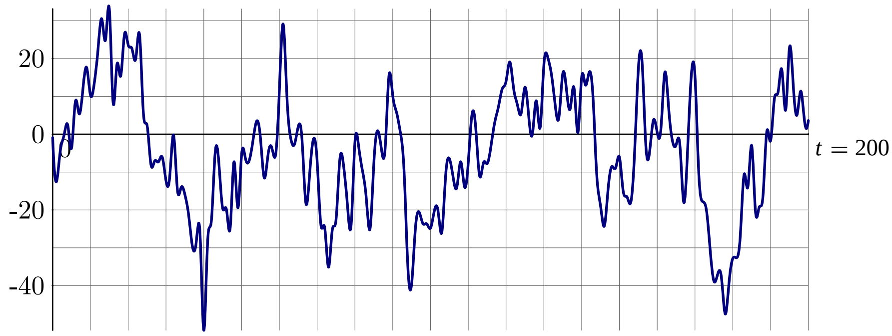
Exam

Time Series Analysis and System Identification (157109)

Date: 10-11-2005
Place: SP-3
Time: 9:00-12:00

Clearly state on your exam which version you are doing: MATH or OTHER (=not math)



1. The above plot depicts a realization $x_0, \ldots, x_{200}$ of a stochastic process X_t. It was generated by *one* of the three schemes below with zero mean, unit variance white noise ϵ_t:

$$X_t = 1.8X_{t-1} + \epsilon_t, \qquad X_t = 0.8X_{t-1} + \epsilon_t, \qquad X_t = 0.8X_{t-1} + 10\epsilon_t.$$

   Explain which one the three was used and why the other two are unlikely.

2. Consider the ARMA(1, 1) scheme

$$(1 - aq^{-1})X_t = (1 - \frac{1}{2}q^{-1})\epsilon_t \tag{1}$$

   with a some real number and ϵ_t a zero mean white noise process with variance σ_ϵ^2.

   (a) For which $a \in \mathbb{R}$ is the scheme stable?

   (b) For which $a \in \mathbb{R}$ is the scheme invertible?

   (c) Determine the 1-step predictor scheme (assuming (1) is stable).

   (d) Determine the 2-step predictor scheme (assuming (1) is stable).

   (e) For some $a \in \mathbb{R}$ the predictors are zero. Explain in words why this is not a surprise.

3. Suppose we have N observations $\epsilon_0, \ldots, \epsilon_{N-1}$ of a zero mean, normally distributed white noise process ϵ_t. Determine the maximum likelihood estimator of the variance of this process.

4. The course does not cover parametric estimation of spectral densities, but this can be done: Propose a method using AR-models to estimate spectral densities based on (as usual) N samples $x_0, \dots, x_{N-1}$ of a wide sense stationary process.

5a. **MATH ONLY:** Subsection 5.2.2 shows how to setup the Least-Squares matrix equations $X = W\theta + \epsilon$ (Eqn. (5.3)) if we want to minimize the cost function (5.2). This cost function has the disadvantage that all model errors $X_t - \mu - a_1 X_{t-1} - \cdots - a_n X_{t-n}$ are considered equally important. To accommodate for model changes it is often better use larger weights for current model errors and smaller weights for errors in the past. This is achieved in the cost function

$$\sum_{t=n}^{N-1} [\lambda^{N-1-t}(X_t - \mu - a_1 X_{t-1} - \cdots - a_n X_{t-n})]^2$$

for some "forgetting factor" $0 < \lambda < 1$. Set up the appropriate Least-Squares matrix equations $X = W\theta + \epsilon$ for minimization of the above cost function and then show that for $n = 0$ the estimated μ becomes

$$\hat{\mu} = \frac{x_{N-1} + \lambda x_{N-2} + \cdots + \lambda^{N-1} x_0}{1 + \lambda + \cdots + \lambda^{N-1}}.$$

5b. **OTHERS (NOT-MATH) ONLY:** Consider the system of Fig. 6.1 [lecture notes page 125] and assume that U_t and V_t are uncorrelated stochastic process, both zero mean and wide sense stationary and that y_t satisfies Eqn. (6.1). What is your estimate of the impulse response h_t if the covariance functions are estimated as

$$\hat{r}_u(k) = \begin{cases} 2 & \text{if } k = 0 \\ 0 & \text{elsewhere} \end{cases}, \qquad \hat{r}_{yu}(k) = \begin{cases} 1 & \text{if } k = 0 \\ 2 & \text{if } k = 1 \\ 0 & \text{elsewhere} \end{cases}.$$

6. Compute the variance of X_t of Problem 2 of this exam for the case that $a = \frac{1}{4}$.

problem:	1	2(a)	2(b)	2(c)	2(d)	2(e)	3	4	5	6
points:	4	2	2	2	2	2	4	3	4	3

The exam grade e is $e = 1 + 9p/p_{\max}$ with p the total score and $p_{\max} = 28$.