Time Series Analysis (& SI)—191571090

(Lecture notes are NOT allowed. For selected formulas see page 3)

Date: 09-11-2012
Place:
Time: 08:45–11:45

1. True or false (and explain your answer):
   (a) Let $X_t = \epsilon_{-2t+1} + \epsilon_{2t}$ with ϵ_t white noise. Is X_t wide-sense stationary?
   (b) Suppose X_t is not white. If the covariance function $r_X(\tau)$ is zero for all lags τ greater than 10, then X_t is not an AR-process?
   (c) The cross covariance function $r_{yu}(\tau)$ of two jointly wide-sense stationary processes U_t, Y_t is even: $r_{yu}(\tau) = r_{yu}(-\tau)$?

2. Let $a, b \in \mathbb{R}$. Suppose ϵ_t is a white noise process with mean zero and variance σ_ϵ^2. Let

$$(1 - aq^{-2})X_t = (1 + bq^{-2})\epsilon_t.$$

   (a) For which a, b is X_t asymptotically wide-sense stationary.
   (b) For which a, b is this scheme invertible?
   (c) Determine the one-step ahead predictor of X_t (you may assume that a, b is such that the scheme is stable and invertible.)
   (d) Determine the two-step ahead predictor of X_t (you may assume that a, b is such that the scheme is stable and invertible.)
   (e) How would you modify the one-step ahead predictor if the mean of ϵ_t is a nonzero μ?
   (f) Let $a = 1/10$. For what values of b is the spectral density $\phi_X(\omega)$ nonzero at every frequency?

3. What is the definition of a *consistent estimator*?

4. Consider the non-stationary process

$$X_t = \frac{\theta}{\sqrt{t}} + \epsilon_t, \qquad t = 1, 2, \ldots \tag{1}$$

   where ϵ_t is standard white noise with $\mathbb{E}(\epsilon_t) = 0, \mathbb{E}(\epsilon_t) = 1$.

   (a) Given $X_1, \ldots, X_N$ write (1) in the vector form $X = W\theta + \epsilon$ (i.e. what is W and ϵ? Notice that X, W, ϵ depend on N.)
   (b) Given $X_1, \ldots, X_N$, determine the best linear unbiased estimator $\hat{\theta}_N$ (i.e. the linear unbiased estimator that achieves minimal $\text{var}(\hat{\theta}_N - \theta)$)
   (c) Determine $\text{var}(\hat{\theta}_N)$ of the above $\hat{\theta}_N$.
   (d) Compute $\lim_{N\to\infty} \text{var}(\hat{\theta}_N)$. (Hint: $\sum_{n=1}^{N} 1/n > \log_2(N)$.)

5. Let X_t be a wide-sense stationary process X_t. The lecture notes mentions two reasons why we prefer the standard biased estimator

$$\hat{r}_N(\tau) = \frac{1}{N}\sum_t (x_{t+\tau} - \hat{m})(x_t - \hat{m})$$

over the standard unbiased estimator

$$\hat{r}_N(\tau) = \frac{1}{N-|\tau|}\sum_t (x_{t+\tau} - \hat{m})(x_t - \hat{m}).$$

What are these two reasons?

6. Suppose X_t is an iid zero mean normally distributed white noise process. How long should we measure before the estimate $\operatorname{var}(\hat{r}_N(0))$ has a standard deviation of less than 0.001 of $r(0)$? [that is: what is the minimal N required?]

   Suppose next that Z_t is also zero mean normally distributed with the same variance as the above X_t but $r_Z(\tau) > 0$ for all τ. Does it take longer (i.e. does it require larger N) before the same accuracy is obtained?

7. Consider the ARX scheme

$$Y_t = (c_0 + c_1 q^{-1})U_t + W_t.$$

   Suppose we have $(u_0, \ldots, u_{N-1})$ and $(y_0, \ldots, y_{N-1})$ for identification.

   (a) Write down the least squares solution of c_1, c_2 in matrix form (you do not have to invert the matrix involved).

   (b) In system identification one says that the input of this ARX scheme is "persistently exciting" if all entries of the two-by-two matrix

$$\left(\frac{1}{N}\begin{bmatrix} u_1 & u_0 \\ u_2 & u_1 \\ \vdots & \vdots \\ u_{N-1} & u_{N-2} \end{bmatrix}^T \begin{bmatrix} u_1 & u_0 \\ u_2 & u_1 \\ \vdots & \vdots \\ u_{N-1} & u_{N-2} \end{bmatrix}\right)^{-1}$$

   are bounded independent of N. Find a nonzero input u_t that is not persistently exciting and argue that it is not a wise input to take.

problem:	1	2	3	4	5	6	7
points:	3+3+3	2+1+3+2+2+1	3	2+2+2+2	3	3	2+3

Exam grade is $1 + 9p/p_{\max}$. (Final grade may depend on homework.)