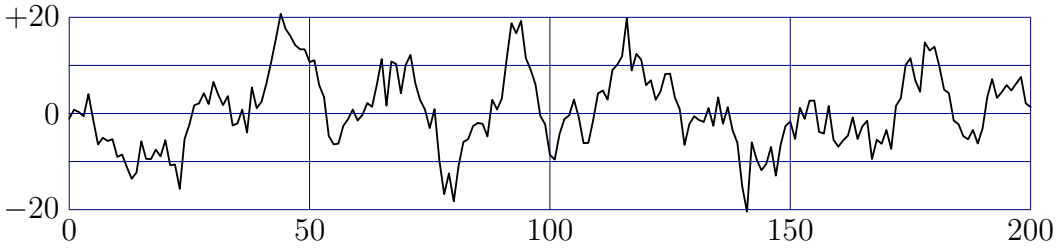
Time Series Analysis (& SI)—191571090

(Lecture notes are NOT allowed. For selected formulas see page 3)

Date: 31-01-2013
Place: CR-3F
Time: 13:45–16:45



1. Three questions

   (a) The above figure shows a realization of 200 samples of a WSS process X_t. It was genered by a first-order scheme $(1 - aq^{-1})X_t = \epsilon_t$ with ϵ_t white noise with $\mathbb{E}(\epsilon_t) = \mu$ and $\text{var}(\epsilon_t) = \sigma^2$. Based on the realization, which values of a, μ, σ are "reazonable"? (A rough estimate is okay, as long as you explain why they are reasonable)

   (b) A WSS process X_t has covariance function $r_X(\tau)$. Express the expected value of $(3X_{t+1} - 2X_t - X_{t-1})^2$ in terms of r_X.

   (c) Is the cross spectral density $\phi_{yu}(\omega)$ of two jointly WSS processes real-valued?

2. Let $b \in \mathbb{R}$. Suppose ϵ_t is a white noise process with mean zero and variance σ_ϵ^2. Let

   $$(2 - q^{-1} + q^{-2})X_t = (b + q^{-1})\epsilon_t.$$

   (a) For which b is X_t is asymptotically wide-sense stationary.

   (b) For which b is this scheme invertible?

   (c) Determine the one-step ahead predictor of X_t (you may assume that b is such that the scheme is stable and invertible.)

   (d) Determine the two-step ahead predictor of X_t (you may assume that b is such that the scheme is stable and invertible.)

   (e) Determine the spectral density of $X_{t+1} - \hat{X}_{t+1|t}$

3. Let $\theta \in \mathbb{R}^m$ and $W \in \mathbb{R}^{N\times m}$. Suppose that the joint density function of

$$X := \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}$$

is

$$f(x) = \frac{1}{(\sqrt{2\pi}\sigma)^N} \mathrm{e}^{-\frac{1}{2\sigma^2}\|X-W\theta\|^2}.$$

   (a) Determine the Maximum-likelihood estimator of both θ and σ.

   (b) Is $\hat{\theta}$ unbiased?

   (c) Is $\hat{\sigma}$ unbiased?

   (d) Formulate the theorem of Cramer-Rao.

   (e) Is $\hat{\theta}$ efficient?

4. Suppose X_t is an iid zero mean normally distributed white noise process. How long should we measure before the estimate $\hat{r}_N(0)$ has a standard deviation of less than $0.01 \times r(0)$? [that is: what is the minimal N required?]

5. Windowing functions $w(k)$ are even functions defined on $[-M, M]$ (and are zero for $k \notin [-M, M]$). They are used to modify the estimate of the spectral density function.

   (a) Why are windowing functions even?

   (b) Why is it beneficial to take M "small"?

   (c) Why is it beneficial to take M "large"?

6. Consider $Y_t = \sum_{m=-\infty}^{\infty} h_m U_{t-m} + V_t$ and suppose U_t and V_t uncorrelated processes.

   (a) Prove that $r_y = r_z + r_v$ where $Z_t = \sum_{m=-\infty}^{\infty} h_m U_{t-m}$.

   (b) If the coherence spectrum $K_{yu}(\omega) = 1/2$ for all frequencies ω, which fraction of the power of y is due to v?

   (c) If the coherence spectrum is 1 for $\omega \in [0, 1]$ and zero for $\omega \in (1, \pi]$, in which frequency-band has system identification been succesful?

problem:	1	2	3	4	5	6
points:	3+2+2	1+2+3+2+3	3+2+1+3+2	2	2+2+2	3+2+2

Exam grade is $1 + 9p/p_{\max}$. (Final grade may depend on homework.)