Test Linear Structures 1
201300056: Structures en Models
Monday October 31 2016; 8:45 - 10:15 uur

This test contains 6 problems. **All answers have to be motivated!** A (graphical) calculator can be used only to check your answers.

1. [6pt] $T: V \to W$ is a linear transformation. Prove that $R(T)$ is a subspace of W.

2. A linear transformation $T: V \to V$ is **not** the identity transform I, and is such that $T^2(v) = T(v)$ for all $v \in V$. (Recall that by definition the transformation $T^2(v)$ is the composite transformation $T(T(v))$).

(a) [4pt] Give an example of such transformation T when $V = P_2(\mathbb{R})$ is a space of polynomials of degree at most 2.

In (b), (c), let $\beta = \{1, x, x^2\}$ be the standard basis for $P_2(\mathbb{R})$. Solve **only one** version of (b) and (c), depending on whether you have solved (a).

If you have solved (a):

(b) [4pt] Compute the matrix $[T]_\beta^\beta$ for T in (a).

(c) [4pt] Verify that $\left([T]_\beta^\beta\right)^2 = [T]_\beta^\beta$.

If you have not solved (a):

(b) [4pt] Compute the matrix $[T]_\beta^\beta$ for T defined as $T(f(x)) = xf'(x)$.

(c) [4pt] Verify that $[T^2]_\beta^\beta = \left([T]_\beta^\beta\right)^2$.

3. $T: V \to W$ is one-to-one but not onto. Let $\beta = \{u_1, u_2, \ldots, u_n\}$ be a basis for V.

(a) [6pt] Prove that $T(\beta)$ is linearly independent.

(b) [6pt] Prove that $\dim(W) > \dim(V)$.

4. In (a),(b), the augmented matrix of a linear system $A\mathbf{x} = \mathbf{b}$ is given as follows:

$$\begin{pmatrix} 1 & -1 & 0 & 3 & 2 & 0 \\ 2 & -2 & 1 & 8 & 0 & 4 \\ -1 & 1 & -3 & -9 & 1 & -3 \end{pmatrix}$$

(a) [8pt] Find the solution set for $A\mathbf{x} = \mathbf{b}$.

See other side

(b) [3pt] Find $\text{rank}(A)$. (Motivate your answer!)

(c) [3pt] Let K_H be the solution set of the system $A\mathbf{x} = \mathbf{0}$. Determine K_H and $\dim(K_H)$.

5. [6pt] A and B are $n \times n$ matrices. Prove that if AB is invertible then A and B are invertible.

6. (a) [6pt] Prove that any eigenvalue λ of an $n \times n$ matrix A satisfies

$$\det(A - \lambda I) = 0.$$

(b) [4pt] Find all eigenvalues of the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ -2 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Total: 60pt

grade=([score for the test on Chapter 1] + [score for this test]+10)/10.