Partial Test 1, Lineaire Structures II, 201300057

Date	:	25 november 2016
Place	:	NH-209
Time	:	13.45 – 15.15
Module-coordinator	:	B. Manthey
Instructor	:	H. Zwart

All answers must be motivated.
The use of (Scientific) calculator, formula sheet, or notes is not allowed.

1. Given is the (complex) linear space V spanned by the functions: $\{e^{ix}, xe^{ix}, x, 1\}$. On this space we consider the linear mapping

   $$T(f) = f' + (1-i)f.$$

   (a) Determine eigenvalues en eigenvectors of T.

   (b) Is T diagonalizable?

   (c) Is T^{-1} diagonalizable?

2. Let T be a linear operator from V to V, and let $W \subset V$ be a linear subspace of V which is T-invariant. We denote by T_W the linear operator T restricted to W, i.e., $T_W : W \mapsto W$ and $T_W(w) = T(w)$, $w \in W$.

   (a) Prove that W is also T^2-invariant.

   (b) Show that if T is invertible, then the same holds for T_W. Is the converse true as well?

3. Let $S_{2\times2}(\mathbb{R})$ be the linear space consisting of all 2×2 symmetric (real) matrices with the (candidate) inner product

   $$\langle P, Q\rangle = 11p_{11}q_{11} + 12p_{12}q_{12} + 22p_{22}q_{22}. \qquad (1)$$

   (a) Prove that (1) defines an inner product on $S_{2\times2}(\mathbb{R})$.

   (b) Construct an element of $S_{2\times2}(\mathbb{R})$ with length 12 which is orthogonal to $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$

   (c) Does (1) define an inner product on $M_{2\times2}(\mathbb{R})$, the linear space consisting of all 2×2 (real) matrices?

Points [1]

Ex.1		Ex. 2		Ex. 3	
a	6	a	4	a	6
b	3	b	6	b	4
c	4			c	3

[1] Total is 40. You will get 4 points for free