

Kenmerk : TW2016/DWMP/009/ha

Course : **Mathematics A (Euclid)**

Date : September 23, 2016

Time : 13.45 – 14.45 hrs

Motivate all your answers.
The use of electronic devices is not allowed.

1. [3 pt]

Let, for $k \in \{1, 2, 3, 4\}$, the intervals $A_k \subseteq \mathbb{R}$ be given by: $A_k = [(-1)^k k, 5k)$.

Determine

$$\bigcap_{k=1}^4 A_k \quad \text{and} \quad \bigcup_{k=1}^4 A_k.$$

2. [3 pt]

Consider the statements

$$\exists x \forall y (x \leq y); \quad \forall x \exists y (x \leq y)$$

Determine for each of these statements if it is true or false in case $x \in \mathbb{N}, y \in \mathbb{N}$ and in case $x \in \mathbb{Z}, y \in \mathbb{Z}$. Explain your answers!

3. (a) [2 pt]

Let $k \in \mathbb{Z}$. Use the definitions of *even* and *odd* to prove that k cannot be both even and odd.

(b) [3 pt]

Prove with mathematical induction that for all $n \in \mathbb{N}$,

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

4. In this exercise your answers must be numbers; if your answer contains binomial coefficients or factorials, like $\binom{8}{3}$ or $8!$, you must work these out.

How many selections of four letters from the set $\{A, B, C, D, E, F, G, H, I, J\}$ are possible if

(a) [1 pt]

Letters may be chosen more than once (e.g. in $AFBF$).

(b) [1 pt]

Letters may not be chosen more than once and the order in which the letters are selected matters (e.g. $DACF$ is considered different from $FCDA$).

(c) [1 pt]

Letters may not be chosen more than once and the order in which the letters are selected does not matter (e.g. $DACF$ and $FCDA$ are considered identical).

Total: 14 points