Exam: Math β 2, Module 2 AM, EE & AT

Bachelor: AM, EE& AT, EEMCS

Codes	201700140 (AM), 201700091 (AT), 201700088 (EE)
Date:	2 February 2018
Time:	13:45-16:45
Module coordinator:	Jasper de Jong (AM), Marcel Ter Brake (AT), Luuk Spreeuewers (EE)
Instructor	Jan Willem Polderman
Type of test	closed book
Allowed aids	nothing

Course	:	Mathematics β II
Date	:	February 2nd, 2018
Time	:	13:45-16:45

Please provide motivation for all your answers and calculations. The use of electronic devices is not allowed.

1. Let S_n be given by

$$S_n = \sum_{k=1}^{n} \frac{k^2 + kn}{n^3}.$$

   (a) Interpret S_n as a Riemann sum of a function $f(x)$ on the interval $[0, 1]$. Hint: take the partition $P_n = \{0, 1/n, 2/n, \ldots, (n-1)/n, 1\}$ as the starting point for rewriting S_n as Riemann sum and determine the function $f(x)$.

   (b) Now calculate

$$\lim_{n\to\infty} S_n.$$

2. (a) Formulate the Mean Value Theorem for Integrals.

   (b) Calculate the average value of $f(x) = x^3 \sin(x^2)$ on the interval $[0, \sqrt{\pi}]$.

3. Consider the series

$$\sum_{n=1}^{\infty} \frac{n^3}{(n+1)!}.$$

   Investigate whether the series converges.

4. Consider the planar curve γ given by

$$(x(t), y(t)) = (t^2, t^3) \quad 0 \le t \le 2.$$

   Determine the length of γ.

5. Let $f(x, y)$ be given by

$$f(x, y) = x^4 - 6x^2y^2 + y^4$$

   See Figure 1 for an impression of the graph of $f(x, y)$.

   (a) Calculate $\text{grad} f(x, y)$.

   (b) Determine all critical points.

   (c) Determine the nature of the critical points, that is, (local) min/max, saddle point. If the second order test does not yield an answer, then consider $f[x, x]$ and $f[x, 0]$.

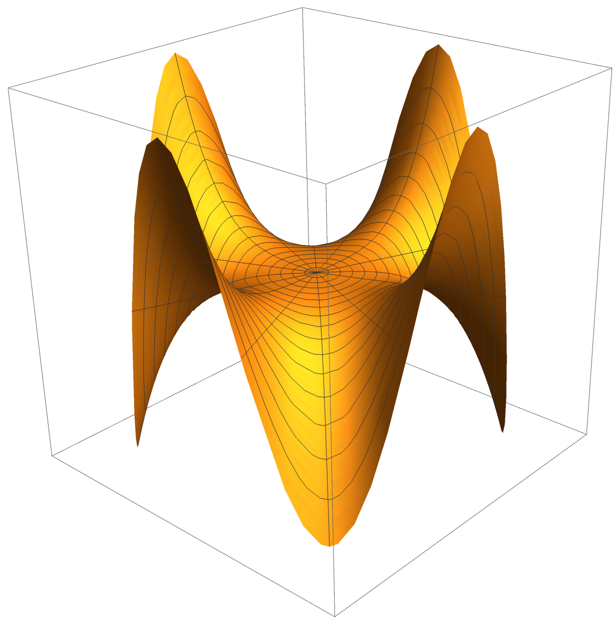

Figure 1: Graph of $f(x, y)$

(d) Use Lagrange multipliers to determine the critical points of $f(x, y)$ on the curve defined by $x^2 + y^2 = 1$.

(e) Determine the nature of these critical points.

(f) Assume that the equation $f(x, y) = -4$ defines, in a sufficiently small, neighborhood of $(x, y) = (1, 1)$, y as a function of x. Determine $y'(1)$.

Points: **Ex 1**, a: 3, b: 4. **Ex 2**: a: 2, b: 4, **Ex 3**: 6 **Ex 4**: 6 **Ex 5**: a: 1, b: 1, c: 2, d: 3, e: 2, f:2.