Kenmerk : TW2014/TW/DWMP/004s/gp

Course : **Mathematics B2: Newton (Solution)**

Date : January 10, 2014
Time : 15.45 - 16.45

1. Partition $[0,1]$ in n subintervals of equal length [$\frac{1}{2}$ pt]

   The function value in the right endpoint $\left(\frac{k}{n}\right)$ is $\left(\frac{k}{n}\right)^4$ [1 pt]

   Corresponding Riemann sum $\sum_{k=1}^{n} \frac{1}{n} \cdot \left(\frac{k}{n}\right)^4$ [1 pt]

   $\sum_{k=1}^{n} \frac{1}{n} \cdot \left(\frac{k}{n}\right)^4 = \sum_{k=1}^{n} \frac{k^4}{n^5}$ [$\frac{1}{2}$ pt]

2. Fundamental theorem: if $F(x) = \int_1^x \frac{e^t}{t} dt$ then $\frac{d}{dx} F(x) = \frac{e^x}{x}$ [1 pt]

   $y = F(\sqrt{x})$, so: [1 pt]

   Using the chain rule $\frac{d}{dx}\left(\int_1^{\sqrt{x}} \frac{e^t}{t} dt\right) = \frac{e^{\sqrt{x}}}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$ [1 pt]

   (The last line alone as solution is OK).

3. (a)

   Substitute $u = \cos(2x)$;

   $x = 0 \rightarrow u = 1$

   $x = \frac{\pi}{6} \rightarrow u = \frac{1}{2}$

   } [1 pt]

   $\frac{du}{dx} = -2\sin(2x)$ so $\sin(2x)dx = -\frac{1}{2} du$ [$\frac{1}{2}$ pt]

   $\int_0^{\frac{\pi}{6}} \frac{\sin(2x)}{\cos^3 2x} dx = \int_1^{\frac{1}{2}} \frac{-\frac{1}{2}}{u^3} du$ [$\frac{1}{2}$ pt]

   $\int_1^{\frac{1}{2}} \frac{-\frac{1}{2}}{u^3} du = \left[\frac{1}{4} u^{-2}\right]_1^{\frac{1}{2}} = 1 - \frac{1}{4} = \frac{3}{4}$ [1 pt]

   (b) The idea of using partial integration [$\frac{1}{2}$ pt]

Choose $f(x) = \ln(x)$ and $g(x) = \frac{2}{3}x^{\frac{3}{2}}$ [1 pt]

$$\int \sqrt{x}\ln(x)dx = \int \ln(x)d\left(\frac{2}{3}x^{\frac{3}{2}}\right) =$$

$$\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \int \frac{2}{3}x^{\frac{3}{2}}d\left(\ln(x)\right) = \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{2}{3}\int x^{\frac{3}{2}} \cdot \frac{1}{x}dx$$ [$\frac{1}{2}$ pt]

$$= \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \int \frac{2}{3}x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}}$$ [$\frac{1}{2}$ pt]

$+C$ [$\frac{1}{2}$ pt]

4. $\displaystyle\sum_{k=1}^{\infty}\frac{x^k}{5^k} = \sum_{k=1}^{\infty}\left(\frac{x}{5}\right)^k$ is convergent if and only if $-1 < \frac{x}{5} < 1$ [1 pt]

   So interval of convergence is $(-5, 5)$ [$\frac{1}{2}$ pt]

   $\displaystyle\sum_{k=1}^{\infty}\frac{x^k}{5^k}$ is geometric series, first term $\frac{x}{5}$, ratio $\frac{x}{5}$ [1 pt]

   Therefore sum= $\dfrac{\frac{x}{5}}{1-\frac{x}{5}}$ [$\frac{1}{2}$ pt]

5. $f(x) = \ln(1+x)$; $\quad f(0) = \ln(1) = 0$ [$\frac{1}{2}$ pt]

   $f'(x) = \frac{1}{1+x}$; $\quad f'(0) = \frac{1}{1+0} = 1$ [$\frac{1}{2}$ pt]

   $f''(x) = \frac{-1}{(1+x)^2}$; $\quad f''(0) = \frac{-1}{(1+0)^2} = -1$ [$\frac{1}{2}$ pt]

   It follows that $P_2(x) = 0 + \frac{1}{1!}x + \frac{-1}{2!}x^2 = x - \frac{1}{2}x^2$ [$1\frac{1}{2}$ pt]

   (Only Taylor polynomial of order 2 at $x = 0$: $P_2(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2$ instead of the previous line: [1 pt])