Exam Vector Calculus
Module 3 Applied Physics and Applied Mathematics
Bachelor

Codes 201700164-201800137
March 1, 2019, 8.45-10.30

- **All answers must be motivated and clearly formulated.**
- **The use of a calculator is not allowed.**

1. Given the contour $C = C_1 \cup C_2 \cup C_3$ with

   i. C_1: line segment from $(2,0)$ to $(0,0)$.

   ii. C_2: line segment from $(0,0)$ to $(\sqrt{2},-\sqrt{2})$.

   iii. C_3: clockwise circular arc with radius 2 from $(\sqrt{2},-\sqrt{2})$ to $(2,0)$.

   Given the vector field $\mathbf{F}(x,y) = F_1(x,y)\mathbf{i} + F_2(x,y)\mathbf{j}$ with

   $$F_1(x,y) = \log(\sqrt{x^2+y^2}),$$
   $$F_2(x,y) = \frac{1}{(x^2+y^2)^{\frac{1}{4}}}.$$

   a. Is $\mathbf{F}$ a conservative field? Motivate your answer.

   b. Calculate $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.

   c. Use Greens' theorem to calculate $\oint_C F_1(x,y)dx + F_2(x,y)dy$.

2. Given the domain D with

   $$D = \{(x,y,z) \in \mathbb{R}^3 \, : \, x^2+y^2+z^2 \leq 4, \; x \leq 0, y \geq 0, z \geq 0\}.$$

   At the surface S of the domain D the unit outward normal vector is denoted as $\hat{\mathbf{N}}$. Given the vector field

   $$\mathbf{F}(x,y,z) = xyz\mathbf{i} + xyz\mathbf{k}.$$

   a. Calculate $\oiint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS$ using Gauss' theorem.

   b. Use spherical coordinates to calculate $\oiint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS$ without using Gauss' theorem.

3. Given the surface S with

$$S = \{(x, y, z) \in \mathbb{R}^3 : x = \sqrt{y^2 + z^2},\ 1 < x < 2\}.$$

At the surface S the unit normal vector $\hat{\mathbf{N}}$ has a negative x-component.

Given the vector field

$$\mathbf{F}(x, y, z) = z\,y\,\mathbf{i} + y^2\mathbf{j} + \exp(x^2)\mathbf{k}.$$

a. Calculate **curl F**.

b. Calculate $\iint_S \mathbf{curl\,F} \cdot \hat{\mathbf{N}}\,dS$ using Stokes' theorem.

Grading

1: 11	2: 9	3: 7
1a: 2	2a: 4	3a: 2
1b: 4	2b: 5	3b: 5
1c: 5		

total 27+3=30 points