Exam Vector Calculus
Module 3 Applied Physics and Applied Mathematics
Bachelor

Codes 201700164-201900257
April 7, 2020, 8.45-10.45

- **This online exam is held based on the assumption that you will do it yourselves, without the help from other people.**

  By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behavior expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

- **In case we observe significant irregularities this will be reported to the Exam Committee.**

- **All answers must be motivated and clearly formulated.**

  During the exam you are only allowed to use the book: Calculus A Complete Course, R.A. Adams, C. Essex, 9th edition, Pearson.

- **Please write on the first page of your answer sheet:**

  I made this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of this test.

  Sign this statement, and write below your signature your name, student number and study program.

- **Exam Questions:**

1. Given the domain R with

$$R := \{(x,y) \in \mathbb{R}^2 \ : \ 1 \leq \sqrt{x^2 + \tfrac{1}{4}y^2} \leq 2\}.$$

The domain R has a *negatively* oriented boundary consisting of

$$C_1 := \{(x,y) \in \mathbb{R}^2 \ : \ \sqrt{x^2 + \tfrac{1}{4}y^2} = 1\},$$
$$C_2 := \{(x,y) \in \mathbb{R}^2 \ : \ \sqrt{x^2 + \tfrac{1}{4}y^2} = 2\}.$$

Given the vector field

$$\mathbf{F}(x,y) := x\mathbf{i} + \frac{xy^2}{\sqrt{4x^2 + y^2}}\mathbf{j}.$$

   a. Sketch the domain R. Indicate the proper orientation of the boundary of R.

   b. Use Green's theorem to compute $\oint_{C_1 \cup C_2} \mathbf{F} \cdot d\mathbf{r}$.

   c. Calculute $\oint_{C_1 \cup C_2} \mathbf{F} \cdot d\mathbf{r}$ directly.

2. Given the surface S with parametrization

$$\mathbf{r}(u,v) := e^u \cos v\mathbf{i} + e^u \sin v\mathbf{j} + u\mathbf{k},$$

where $0 \leq u \leq 1$, $0 \leq v \leq \pi$. The z-component of the normal vector $\mathbf{N}$ at the surface S is negative.

Given the vector field

$$\mathbf{F}(x,y,z) := yz\mathbf{i} - xz\mathbf{j} + (x^2 + y^2)\mathbf{k}.$$

   a. Calculate the integral
$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

   b. Is the vector field $\mathbf{F}$ conservative? Motivate your answer.

3. Given the surfaces

$$\begin{aligned} S_1 :=& \{(x,y,z)\in\mathbb{R}^3 \ : \ y = x^2+z^2, \ x\geq 0, \ 0\leq y\leq 4\}, \\ S_2 :=& \{(x,y,z)\in\mathbb{R}^3 \ : \ y \geq z^2, \ x = 0, \ 0\leq y\leq 4\}, \\ S_3 :=& \{(x,y,z)\in\mathbb{R}^3 \ : \ x^2+z^2 \leq 4, \ x\geq 0, \ y = 4\}. \end{aligned}$$

At the surface S_1 the unit normal vector $\hat{\mathbf{N}}$ has a negative y-component.
At the surface S_2 the unit normal vector $\hat{\mathbf{N}}$ has a negative x-component.
At the surface S_3 the unit normal vector $\hat{\mathbf{N}}$ has a positive y-component.

The closed surface S is the union of the surfaces S_1, S_2, and S_3, hence $S = S_1 \cup S_2 \cup S_3$.

Given the vector field

$$\mathbf{F}(x,y,z) := zy\mathbf{i} + y^2\mathbf{j} + xy\mathbf{k}.$$

a. Give the parametrization of the surface S_1.

b. Calculate $\iint_{S_1\cup S_2} \textbf{curl } \mathbf{F}\cdot\hat{\mathbf{N}}dS$ using Stokes' theorem. Sketch the surface $S_1 \cup S_2$ and indicate the orientation of the boundary of $S_1 \cup S_2$.

c. Calculate $\iint_S \textbf{curl } \mathbf{F}\cdot\hat{\mathbf{N}}dS$. Explain why you can give this answer directly without any computation?

d. Calculate $\iint_S \mathbf{F}\cdot\hat{\mathbf{N}}dS$.

- Upload a scan of your answers, your student card, and if applicable your card for extra time, **as a pdf file** (preferably one file) in the Assignment Section of Canvas Canvas:

  Mod 03 TN/AM Joint Parts (2019-2A)\Assignments\Retake Vector Calculus April 7

  Other formats than pdf will not be accepted.

  After the end of the exam, or any earlier, you will have 20 minutes for the uploading of your answers into Canvas. After this, the site will be closed and no answers will be accepted. Hence uploading is possible till 11.05 for regular students and 11.35 for students with permission for extra time.

  Please make sure your scan is readable. **What we cannot read, we cannot grade.**

Grading

1: 10	2: 4	3: 13
1a: 2	2a: 3	3a: 2
1b: 4	2b: 1	3b: 5
1c: 4		3c: 2
		3d: 4

total 27+3=30 points