Instructions

You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in.

You may use a hand-written formula sheet containing maximum 10 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (8pts/100)

Figure 1 shows a set of electric field lines **E**. Copy the figure and superimpose a sketch of 5 equipotential surfaces on it. Make the voltage 'steps' identical from surface-to-surface (i.e $V_2=V_1+\Delta V$; $V_3=V_2+\Delta V$ etc.) and indicate in the sketch which surface corresponds to the highest voltage and which to the lowest.

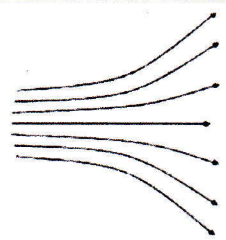

Figure 1: Electric field lines fanning out (problem 1).

Problem 2 (10pts/100)

A uniform **E**-field (5 N/C) points in the +y-direction (figure 2). What is the electric flux through the following surfaces:

2.1 : LMNO **2.2** : MNRP **2.3** : LMP
2.4 : LORP **2.5** : The total surface of the prism

For each of these surfaces, select one of four possible answers:

A) 0; B) 5Nm^2/C; C) 2.5Nm^2/C; or D) None of these.

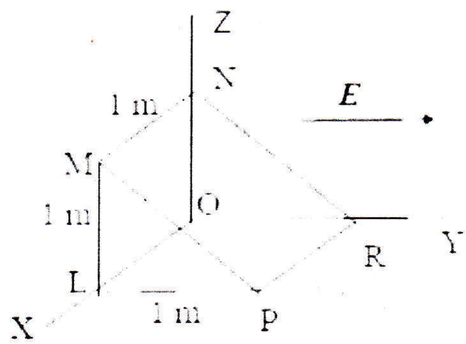

Figure 2: A virtual prism placed in a homogeneous electric field (problem 2).

Problem 3 (16pts/100)

Below you find eight statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

3.1 The curl of the gradient of a scalar field is always equal to zero.

3.2 Electric field lines never cross each other.

3.3 The strength of the electric field near a homogeneously charged infinite flat plane decreases as 1/x with the perpendicular distance x to the plane.

3.4 The electric flux through an equipotential surface always equals zero.

3.5 Free electric charge will distribute itself symmetrically on a conducting sphere, irrespective of the surroundings of the sphere.

3.6 When we place a dielectric material in an electric field, the resulting field inside the dielectric will always be lower than the field outside.

3.7 The potential difference between the plates of a charged and disconnected capacitor remains the same when the space between the plates is filled with a dielectric.

3.8 When there's no free charge present, the electric displacement **D** is continuous (in magnitude and direction) across the interface between two different dielectric materials.

Problem 4 (33pt/100)

Consider a circle in the *xy*-plane, centered on the origin with radius *R*. A charge is homogenously distributed over the circumference of the circle creating a line-charge-density λ [C/m].

4.1 Show that the electric field **E** on the z-axis (thus $\perp$ above the center of the circle) is given by:

$$\mathbf{E} = E_z\,\hat{\mathbf{z}} = \frac{\lambda}{2\varepsilon_0}\frac{zR}{\left(z^2+R^2\right)^{3/2}}\,\hat{\mathbf{z}}\ .$$

4.2 Now, consider a (filled) circular plate in the *xy*-plane with radius R_0, centered on the origin, carrying a homogenous surface charge density σ [C/m²]. Using the result from 4.1, show that the field $\perp$ above the center is given by:

$$\mathbf{E} = E_z\,\hat{\mathbf{z}} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R_0^2}}\right)\hat{\mathbf{z}}\ .$$

4.3 Simplify the equation under 4.2 for the limits $z \ll R_0$ and $R_0 \ll z$, i.e. close-by and far-away from the plate. Comment on the physical significance of these simplified expressions.

Hint *"far-away"*: $(1+u)^n \approx 1+nu$ when $u \to 0$

Problem 5 (33pt/100)

A metal ball with radius R carries a charge q_0 and is coated with a spherical dielectric shell of thickness d and relative permittivity ε_r (Figure 3).

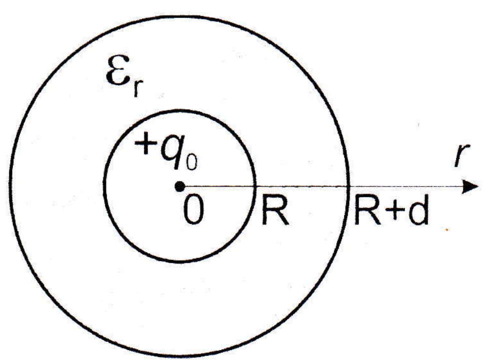

Figure 3: A charge-carrying metal ball surrounded by a dielectric shell (problem 5).

5.1 Work out an expression for the electric field **E** as a function of *r*, the distance to the centre of the ball.

Make a graph of the magnitude of **E** as a function of *r*. The sketch should at least cover the region ($0 \le r \le$ R+3d) and key values of E should be indicated on the vertical axis.

5.2 Work out the electric potential V_0 at the origin as function of the parameters q_0, R, d and ε_r (take the potential at infinity as reference.

5.3 Work out the free surface charge density σ_f (*r*=R) at the surface of the ball; the bound surface charge density σ_b (*r*=R) at the inner surface of the dielectric shell; the bound volume charge density $\rho_b(r)$; and the bound surface charge density σ_b (*r*=R+d) at the outer surface of the shell. (Once more as a function of the parameters q_0, R, d and ε_r. Also give the sign, positive or negative.)