E&M
UNIVERSITY OF TWENTE.

201300164 & 201300183
19th June 2017

Boller, Offerhaus, Dhallé
E&M Test 2
Fields and electromagnetism

ATTENTION: Questions 1–4 need to be made only by students doing the full exam (both the E and M parts, not just M). Questions 5 – 8 need to be made by all students. Questions 9 & 10 need to be made only by students that take the "M-test" (i.e. NOT by the students that do the full exam)

Problem 1 (15points): A metal plate is of thickness d is placed exactly in the middle between two point charges $+q$ and $-q$, which are separated by a distance $3d$ from each other (Fig. 1).

1.1 Copy the figure and sketch the electric field lines **E** in this configuration.

1.2 Add equipotential lines to your sketch.

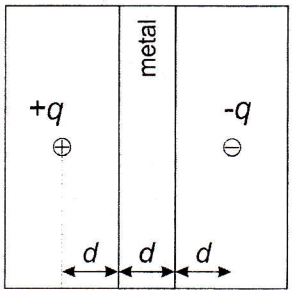


Fig. 1: two point charges with a flat metal plate in-between (problem 1).

Problem 2 (10 points): Below you find five statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

2.1 Inside a homogeneously (positively) charged spherical shell, a positive charge experiences an electric force towards the centre of the sphere.

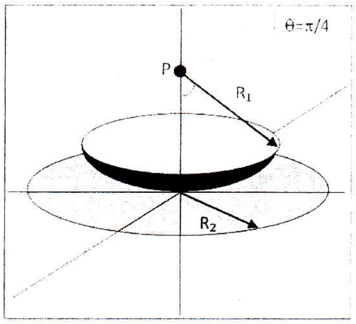
2.2 The total electric flux through the surface of a cube that envelops a charge distribution is the same as the flux trough a sphere that encloses the same charge distribution.

2.3 In order to keep a free positive point charge immobile in an electric field, one has to apply a force along the positive gradient of the potential.

2.4 The work needed to move a charge from position A to position B in an inhomogeneous electric field depends on the path that one follows.

2.5 When placed in an inhomogeneous electric field, a dielectric substance will experience a force in the direction of the divergence of the field.

Problem 3 (25 points): In the image on the right you see a segment of sphere that touches the xy plane in the origin. The sphere is covered with a surface charge density σ. P is the centre of the sphere. Calculate the electric field at some point (z) located on the z-axis above point P.



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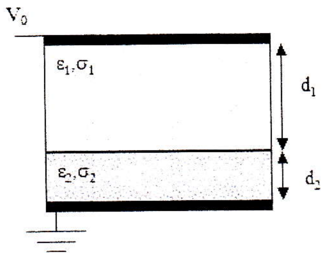
Problem 4 (25 points): The image on the left shows a capacitor that is filled with two different materials where both materials are somewhat conducting (with conductivity σ) and have a dielectric response ε.



The (leaky) capacitor is connected to a voltage source that supplies a constant voltage V_0

4.1 Where is the free charge located and where is the (net) bound charge?

4.2 Express the free surface charge density between the dielectrics in terms of V_0, ε, σ and d.

ATTENTION: Questions 5 – 8 need to be made by all students

Problem 5 (15 points): Three straight wires are arranged parallel to each other, with an equal distance between them. The outside wires carry an amount if current in one direction, the middle wire carries the same amount of current in the opposite direction. Sketch the magnetic field line around the wires. Make sure that you chose an appropriate plane in which to sketch this, indicate the direction of the current and the direction of the field. Make the sketch big enough to show what happens between the wires and far away from (all three) wires.

Problem 6 (10 points): Below you find five statements. For each of them, indicate whether the statement is 'true' (T) or 'not true' (NT). Also include a brief argument why you agree or not (minimum 1 & maximum 5 lines per statement). Read the statements carefully, each word may be important!

6.1 In a magnetic field (**B**) that points in the (Cartesian) +x-direction, a current that flows in the +y-direction will experience a Lorentz force in the +z direction.

6.2 A magnetic field with non-zero divergence would imply the existence of magnetic monopoles.

6.3 When the current in a coil is increased from I_0 to $2I_0$, the magnetic energy stored in the system also doubles.

6.4 An object with magnetization **M** is moved inside an infinitely long current-carrying coil, along its axis. The direction of motion determines whether this costs or releases energy.

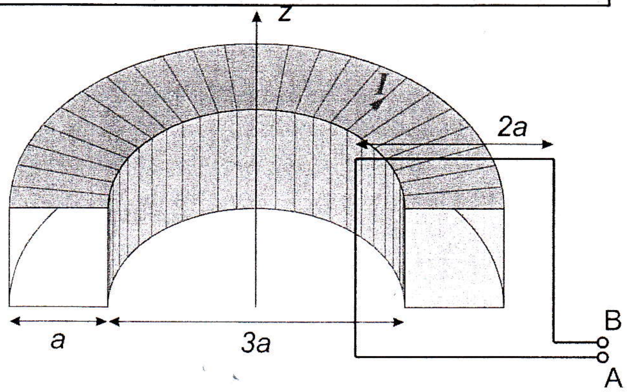
6.5 The electromotive force (e.m.f.) induced in a circuit by a changing current in a second circuit is identical to the e.m.f. that would be induced in the 2nd circuit if the current in the 1st circuit would be changing with the same rate.

Problem 7 (25 points): Calculate the magnetic field at a distance d above one endpoint of a horizontal finite line-segment of length L that carries a current I.

Problem 8 (25 points): An empty toroidal coil with square cross section of side a and inner diameter $3a$ is wound with N turns of wire and carries a current I (Figure on the right).

8.1 Show that the magnetic field **B** *inside* the coil can be written as $\mathbf{B} = \frac{\mu_0 NI}{2\pi}\frac{1}{s}\hat{\boldsymbol{\varphi}}$ (in cylindrical coordinates). What is the field outside the coil?

8.2 A single square wire loop (with side $2a$) is placed symmetrically around the coils cross-section (see figure). What is the magnetic flux ϕ_m through this loop (express your answer in terms of a, N and I)?



Toroidal coil with pick-up loop (only ½ the coil is shown) (problem 8).

8.3 Take a = 1cm and N = 250. Starting from zero, the current I is ramped up with a steady rate of 0.5A/s to a maximum value of 5A and then ramped back down to zero with the same (but negative) rate. Sketch the voltage $V_{AB} = V_B - V_A$ measured over the wire loop from 8.2 as a function of time. Put numbers with the horizontal and vertical axes.

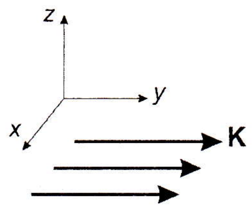
8.4 The toroidal coil is now totally filled with a linear magnetic material with susceptibility χ_m= 4 and the experiment described under 8.3 is repeated. What will be the maximum observed voltage this time?

ATTENTION: Questions 9 & 10 need to be made only by students that take the "M-test" (i.e. NOT by the students that take the full exam)

Problem 9 (10 points): An infinite flat plate is taken as the *xy*-plane of a Cartesian coordinate system. The plate carries a homogeneous surface current **K** in the positive *y*-direction (Figure on the right).

9.1 Copy the figure and sketch a vector representation of the **B**-field at 2 different heights $+z_0$ and $+2z_0$ above the plate, as well as at heights $-z_0$ and $-2z_0$ below the plate.

9.2 Do the same thing for the vector potential **A**.



current-carrying flat plate (problem 9).

Problem 10 (15 points): A long and straight cylindrical conductor of radius a caries a total current I homogeneously distributed over its cross-section. It is tightly enveloped by a diamagnetic cylindrical mantle of thickness a which has a magnetic susceptibility χ_m = -1/2.

10.1 Work out an expression for the magnetic induction **B** inside the conductor ($s < a$), inside the mantle ($a < s < 2a$) and outside the mantle ($2a < s$).

10.2 What are the bound surface currents $\mathbf{K}_b$ at $s = a$ and at $s = 2a$?

10.3 What is the bound volume current $\mathbf{J}_b$ inside the mantle?