Partial test I E&M
22 May 2018

Offerhaus, Marpaung, Dhallé

2017-201400535-2B

Fields and electromagnetism

Instructions

You have **2 hours** to complete the test. Clearly indicate your name and student number on every sheet that you hand in.

You may use a hand-written formula sheet containing maximum 10 equations. This sheet must be handed in together with your answers.

Before answering the questions, read all of them and start with the one you find easiest.

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (12pts/100)

Consider two homogeneously charged concentric spherical shells (figure 1). The inner shell has a radius R_1 and carries a uniform surface charge $\sigma_1 = +q/4\pi R_1^2$. The outer shell has a radius R_2 and carries a uniform charge density $\sigma_2 = -2q/4\pi R_2^2$.

1.a. Copy the figure and sketch the **E**-field lines inside the inner shell; in-between the two shells; and outside the outer shell. Make sure that you represent both magnitude and direction of the field correctly.

1.b. In the same figure, sketch a family of equidistant equipotential surfaces (‘*equidistant* ’ referring to the potential difference ΔV between two successive surfaces). Choose the potential to be zero on the outer shell. Clearly indicate also the sign of the potential at each line (positive or negative).

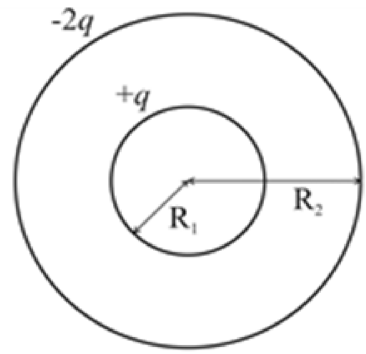

Figure 1: charged concentric spherical shells (problem 1).

Problem 2 (12pts/100)

The space in-between the plates of a planar capacitor is entirely filled with two different layers of homogeneous dielectric material. The thicknesses of these layers are d_1 and d_2 ; their dielectric constants are ε_{r1} and ε_{r2} , with $\varepsilon_{r1} < \varepsilon_{r2}$.

2.a. Sketch a cross-section of such a capacitor and indicate in this sketch the location of all charge (free and bound). Sketch also the vector fields **E** and **D** in each layer, paying attention to the relative magnitude of the vectors.

2.b. Show that the capacitance value C of such a capacitor can be written as

$$C = A\varepsilon_0 \frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1}d_2 + \varepsilon_{r2}d_1} ,$$

with A the surface area of each of the plates.

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Problem 3 (16pts/100)

Below you find eight statements. For each of them, indicate whether the statement is ‘true’ (T) or ‘not true’ (NT). Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

3.a. Throughout the whole of space, the **E**-field generated by an electrically charged sphere with a spherically symmetric charge distribution $\rho(r)$ is identical to the field of a point charge with the same total charge placed at its centre.

3.b. The total electric flux through the surface of a cube with side a that fully encloses a charge distribution $\rho(\boldsymbol{r})$ is larger than the flux through the surface of a sphere with diameter a that fully encloses the same charge distribution, because the surface area of the cube is larger.

3.c. In order to keep a positive test-charge from moving in an electric field, one has to exert a force that points in the direction of the gradient of the potential.

3.d. If A and B are two points that lie close together but *not* on the same equipotential surface, the electric field strength in A and B has to be different.

3.e. The work that needs to be done to move a charge in an non-uniform electric field from a point A to a point B does not depend on the path that is followed.

3.f. When one pulls the charged plates of an extended parallel-plate capacitor apart (with the capacitor not attached to a voltage source), the voltage difference between the plates will increase.

3.g. Far away from an electrical dipole, the electric field strength on its axis decreases as $1/r^3$ (with r the distance to the dipole).

3.h. At the transition from a polarized dielectric (without free charge) to the vacuum outside, both the components of **D** perpendicular and parallel to the surface are continuous.

Problem 4 (30pts/100)

4.a A straight line segment with length L carries a uniform charge density $+\lambda$ (figure 2.a). Show that the electric field in a point P at a perpendicular distance s above the centre of the segment is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{s\sqrt{s^2+(L/2)^2}}\,\hat{\mathbf{s}}$$

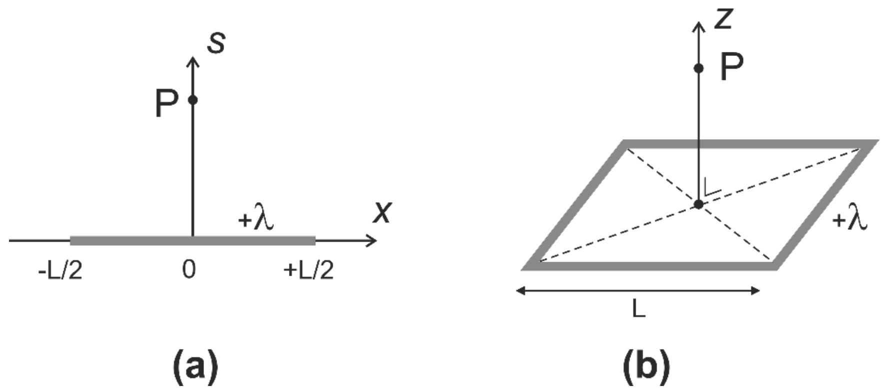

Figure 2: (a) homogeneously charged line segment and (b) charged square wire-frame built up from four such segments (problem 4).

4.b Use this result to derive an expression for the electric field **E** at a height z above the centre of a square wire-frame with sides L that carries a homogeneous charge density $+\lambda$ (figure 2.b). Make clear sketches to illustrate the angles, distances, etc… that you use.

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Problem 4 (30pts/100)

A long, straight and solid cylindrical *conductor* of length L with radius R_1 carries a total electric charge +Q. Concentric around this central rod, there's a thick-walled hollow *conducting* cylindrical sleeve, also of length L, with inner radius $R_2 > R_1$ and outer radius $R_3 > R_2$, which carries an opposite charge –Q (figure 3, left).

The space in-between the rod and the sleeve is filled with dry air, which has an electrical permittivity $\varepsilon \approx \varepsilon_0$. The length L of the system is much larger than its outer radius ($L >> R_3$), so that end-effects can be ignored and the system can be treated as infinitely long in the axial direction.

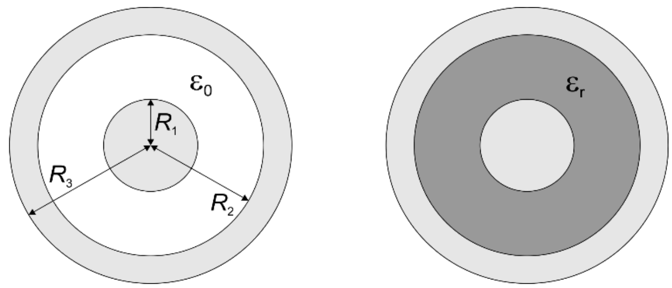

Figure 3: cross-sectional view of a cylindrical capacitor without (left) and with (right) dielectric (problem 5).

5.a. What's the volumetric charge density ρ_R inside the central rod and ρ_S inside the wall of the outer cylindrical sleeve? What is the surface charge density σ_1 on the surface of the rod (at $s = R_1$) and the surface charge densities σ_2 and σ_3 on the inner- and outer surfaces of the sleeve (i.e. at $s = R_2$ and $s = R_3$, respectively)?

5.b. What's the electric field **E** inside the ($s < R_1$); in-between the rod and the sleeve ($R_1 < s < R_2$); inside the wall of the sleeve ($R_2 < s < R_3$); and outside the sleeve ($R_3 < s$)?

5.c. What's the potential difference ΔV between the rod and the sleeve?

5.d. What's the capacitance value C_0 of this cylindrical capacitor?

Next the space in-between rod and sleeve is fully filled with a dielectric material with relative permittivity ε_r . The free charges +Q and –Q on rod and sleeve remain the same (figure 3, right).

5.e. What's in this new situation the electric field **E** in-between the rod and the sleeve?

5.f. What's the ratio C / C_0 between the new capacitance value C and the original value C_0?

5.g. What's the bound surface charge σ_{b1} on the inside of the dielectric (at $s = R_1$) and σ_{b2} on its outside surface (at $s = R_2$). What's the volumetric bound charge density $\rho_b(s)$ inside the dielectric?