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E&M Re-Test 2
April 14th 2020

201300164 & 201300183
Fields and electromagnetism

Instructions

You have **2 hours** to complete the test and 15 minutes to submit. Clearly indicate your name and student number on the pdf file that you submit. If you have additional time please upload an image of that card with your exam. You may use a hand-written formula sheet containing maximum 10 equations. You may use the book (Griffiths). You may NOT use your notes or answer to exercises.
Please start a new page for a new exercise, Upload all pages at the end in a single pdf file..

The amount of points to be obtained with each question is indicated next to the question number.

Problem 1 (15 points/100) Sketch the magnetic field lines for 3 different situations:

1.a. For a current carrying ring of radius *R*. Sketch the field near the ring (*r* up to ~*3R*) and far away (*r>>R*)

1.b. For a short coil (length *L* on the same order as radius *R*). Sketch this for r up to ~ *3R*.

1.c. For a flat ribbon (width *W*, thickness *d~W/10*, length *L~5W*), Sketch for r up to ~*2L*.

Problem 2 (16 points/100)**.** Below you find eight statements. For each of them, indicate whether the statement is 'true' **(T)** or 'not true' **(NT)**. Also include a brief argument why you agree or not (**minimum** 1 & **maximum** 5 lines per statement). Read the statements carefully, each word may be important!

2.a. In the Thompson experiment, where a charged particle is injected with a velocity **v** in a space that contains both an electric **E-** and a magnetic **B**-field, the trajectory of the particle can be made straight only if **E** and **B** are mutually perpendicular;

2.b. The magnetic vector potential in the vicinity of a straight current-carrying wire circles around the wire;

2.c. In a vacuum (a part of space without materials, particles present), any static magnetic field will be rotation-less.

2.d. When one doubles the radius of a long solenoid but keeps the number of windings per unit length and the current in the windings constant, the energy stored in the coil quadruples;

2.e. The bound surface current on the mantle of a diamagnetic cylinder that is coaxially placed inside a long solenoid runs in the same direction as the free current in the solenoid;

2.f. The total electrical power P converted into heat by a current running through a conducting material depends on the electrical resistivity of the material, but not on the size of the conductor;

2.g. The emf that is induced over a wire loop that rotates in a homogeneous magnetic field depends on the orientation of the rotation axis with respect to the field;

2.h. The self-inductance L of a current-carrying circuit is proportional to the current that flows through the circuit.

Problem 3 (14 points/100)

3.a. Two infinitely long, thin and straight conductors are running parallel to each other and are placed a distance *d* = 1 cm apart. Each conductor carries a current *I* = 100 A, but in opposite directions. Work out the force per unit length between the conductors (in N/m). Is this force repulsive or attractive?

3.a. Two infinite, thin and flat conducting planes are placed parallel to the xy-plane, a distance Δz =10 cm apart. Each plate carries a surface current density *K* = 1000 A/m, one plate in the *+x* and the other one in the *-x* direction. Work out the magnetic pressure between the plates (the force per surface area, in units N/m^2).

Problem 3 (30 points/100)**.** A long thick conducting wire with radius R is centered in the z-axis. The wire carries a current I that is homogenously spread over the crosssection surface. The material has a relative magnetic susceptibility of μ_r. The wire may be considered infinitely long (edge effects can be ignored).

3.a. Derive an expression for the volume current density J.
3.b. Derive expressions for the magnetic field **H** inside the wire AND outside the wire. Indicate both strength and direction.
3.c. Derive expressions for the magnetic induction **B** inside the wire AND outside the wire as well as for the magnetization **M**. Indicate both strength and direction.
3.d. Derive expressions for the bound current (surface $\mathbf{K_b}$ and volume $\mathbf{J_b}$). Indicate both strength and direction.
3.*d.* Derive and expression for the magnetic vector potential inside AND outside the wire (choose a zero point yourself). Again show the direction and amplitude.
3.e. Assume that μ_r=2, sketch the relevant component as a function of r.

Problem 4 (25 points/100)**.** A long coil of radius R_1 is centered on the z axis where the wire is looped with *n1* turns per meter length and such that the current (at $t=0$) runs in the $+\phi$ direction. Another loop of radius R_2 ($R_2=2R_1$) with n_2 turns per meter ($n_2=4n_1$) is wrapped around the first coil in the same direction. The voltage over a length of one meter in the inner coil is given by $V_1=V_0 \cos(\omega t)$. Coil_1 has a resistance of $R_{\Omega 1}$, Coil_2 has a resistance of $R_{\Omega 2}$.

4.*a.* Calculate the magnetic induction ***B*** as a function of *r* (inside R_1, and outside R_1, ignoring any current in the second coil for now). The number of turns N is high enough for the coil to be wound tightly so that stray fields and edge effects may be ignored.

4.*b.* Calculate the induced voltage in the outer loop per unit length and the induced current. You may ignore back-action on coil_1 for now.

4.*c.* Calculate the flux that is generated by coil_2 in coil_1 and calculate the induced voltage in coil_1.

4.*d.* Derive an expression for the total voltage in coil_1 (external and induced).

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint \mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint \mathbf{A}\cdot d\mathbf{l}$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$	(permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$	(permeability of free space)
$c = 3.00 \times 10^{8}\,\mathrm{m/s}$	(speed of light)
$e = 1.60 \times 10^{-19}\,\mathrm{C}$	(charge of the electron)
$m = 9.11 \times 10^{-31}\,\mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Standaardintegralen .

$I = \int x^m (a^2+x^2)^n\,dx$ Noem $Y = \sqrt{a^2+x^2}$; $Y^2 = a^2+x^2$;

m	n	I	m	n	I
-2	-1/2	$-Y/(a^2x)$	1	-3/2	$-1/Y$
-2	-1	$-a^{-2}(\frac{1}{x}+\frac{1}{a}\arctan\frac{x}{a})$	1	-1	$\ln\lvert Y\rvert$
-1	-3/2	$a^{-2}(\frac{1}{Y}-\frac{1}{a}\ln\left\lvert\frac{a+Y}{x}\right\rvert)$	1	-1/2	Y
-1	-1/2	$-(1/a)\ln\lvert(a+Y)/x\rvert$	1	1/2	$\frac{1}{3}Y^3$
-1	-1	$a^{-2}\ln\lvert x/Y\rvert$	1	3/2	$\frac{1}{5}Y^5$
0	-3/2	$x/(a^2Y)$	2	-3/2	$\ln\lvert x+Y\rvert - x/Y$
0	-1	$a^{-1}\arctan(x/a)$	2	-1	$x - a\arctan(x/a)$
0	-1/2	$\ln\lvert x+Y\rvert$	2	-1/2	$\frac{1}{2}xY - \frac{1}{2}a^2\ln\lvert x+Y\rvert$
0	1/2	$\frac{1}{2}xY + \frac{1}{2}a^2\ln\lvert x+Y\rvert$	2	1/2	$\frac{1}{8}x(2x^2+a^2)Y - \frac{1}{8}a^4\ln\lvert x+Y\rvert$
0	3/2	$\frac{1}{8}x(2x^2+5a^2)Y + \frac{3}{8}a^4\ln\lvert x+Y\rvert$	3	-3/2	$Y + a^2/Y$
			3	-1/2	$\frac{1}{3}Y^3 - a^2Y$
			3	1/2	$\frac{1}{5}Y^5 - \frac{1}{3}a^2Y^3$

$I = \int \sin^m ax \cos^n ax\,dx$

m	n	I	m	n	I
1	0	$-(1/a)\cos ax$	1	1	$(\sin^2 ax)/2a$ of $-(\cos^2 ax)/2a$
0	1	$(1/a)\sin ax$	2	2	$-\frac{1}{32a}\sin 4ax + \frac{x}{8}$
1	-1	$-(1/a)\ln\lvert\cos ax\rvert$	1	n	$-\frac{\cos^{n+1} ax}{(n+1)a}$
-1	1	$(1/a)\ln\lvert\sin ax\rvert$	m	1	$\frac{\sin^{m+1} ax}{(m+1)a}$
2	0	$\frac{1}{2}x - \frac{1}{4a}\sin 2ax$	0	2	$\frac{1}{2}x + \frac{1}{4a}\sin 2ax$
3	0	$-\frac{1}{3a}\cos ax(\sin^2 ax + 2)$	0	3	$\frac{1}{3a}\sin ax(\cos^2 ax + 2)$
4	0	$\frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$	0	4	$\frac{3x}{8} + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$

6.5 Benaderingen voor $|x| \to 0$

$(1+x)^a$	$1+ax+\dots$	$\sin x$	$x - x^3/6+\dots$
e^x	$1+x+\dots$	$\cos x$	$1 - x^2/2+\dots$
$\ln(1+x)$	$x - x^3/2+\dots$		