Test 1 for Probability Theory
(Module Signals and Uncertainty, 201300182)
Friday March 3, 2017, 13.45 - 15.15 uur

This test consists of 4 problems. Motivate all answers. Using a calculator is *not* allowed.

1. Consider a probability space (S, P).

   a. Formulate Kolmogorov's axioms (i)-(iii) for the probability $P(\cdot)$.

   b. Use these axioms to prove the following: for any event $E \subset S$ it holds that $P(E^c) = 1 - P(E)$.

2. Claire and David ask a friend to vote for the elections on behalf of (both of) them. Together they flip a fair coin once, to decide whether Ali or Bert will vote for them. When the outcome is heads, Ali selects two candidates at random from the list of DENK; this is done with replacement, since both votes may go the same candidate. When the outcome is tails, Bert selects two candidates in the same way, but from the list of VNL. Both lists have 18 candidates, DENK having 7 women and 11 men, while VNL has 2 women and 16 men.
   Define and use proper events to answer the following questions.

   a. When Claires vote goes to a woman, what is the probability that Ali voted?

   b. When both votes go to a woman, then what is the probability that Ali voted?

   c. Now suppose the coin is unfair and has a probability p of coming up heads. How large is p when the probability that Claires vote goes to a woman is $1/3$?

3. The random variable X has the following probability mass function.

$$p(k) = c\,\frac{3^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

   a. Determine the value of c.

   b. Determine the probability that $X^2 + 2X \geq 8$.

   c. Give the definition (formula) for the expectation of a discrete random variable, and use this to derive the expectation of the random variable X.

4. The random variable X has a uniform distribution on the interval $(0, 2)$.

   a. Determine the expectation of X and the variance of X.

   b. Determine the expectation of e^X.

   c. Determine the probability density function of the random variable $Y = e^X$.

Norm:

1a	1b	2a	2b	2c	3a	3b	3c	4a	4b	4c	Total
2	2	3	3	2	1	3	3	2	2	4	27

Grade: $\dfrac{\text{total}}{3} + 1$ (this grade is 30% of the final grade for Probability Theory.)