UNIVERSITY OF TWENTE

Department of Electrical Engineering, Mathematics and Computer Science

Exam Signals and Transforms on Thursday March 9, 2017, 8.45 – 10.15 uur.

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer!

You are not allowed to use a calculator. Besides pen and paper, the only thing you are allowed to use is one handwritten, singe-sided, A4-sized page of personal notes.

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1. Let $f(t)$ be the 2-periodic function which satisfies:

$$f(t) = \mathbb{1}(t-1), \qquad \text{for } t \in [0,2)$$

   a) Sketch the function $f(t)$ for $t \in [-5,5]$.

   b) Determine the complex Fourier series for $f(t)$.

   Let $g(t)$ be the $\frac{\pi}{2}$-periodic function which satisfies:

$$g(t) = \cos t, \qquad \text{for } t \in [0, \tfrac{\pi}{2})$$

   whose complex Fourier coefficients are equal to:

$$g_k = \frac{2+8ki}{(1-16k^2)\pi}$$

   c) Determine the real Fourier series for $g(t)$.

   d) Is g equal to the real Fourier series for all $t \in \mathbb{R}$?

   e) Determine the generalized derivative of the function g.

2. Show that the convolution of $f(t) = \cos(\pi t)$ and $g(t) = e^t\,\mathbb{1}(1-t)$ is equal to:

$$\frac{e}{1+\pi^2}\left[\pi\sin(\pi t) - \cos(\pi t)\right]$$

see reverse side

3. Consider the space $\mathcal{L}^2[-\pi,\pi]$ and the subspace $\mathcal{L}^{2,\text{even}}$ of even functions.

   a) Assume f is an even and g is an odd function. Show that f and g are orthogonal with respect to the standard inner product.

   b) Assume f is an odd function. Show that 0 is the best approximation of f in the space $\mathcal{L}^{2,\text{even}}$.

   c) Consider the following three functions:

$$f_1(t) = \frac{1}{\sqrt{\pi}}\cos(t)$$
$$f_2(t) = \frac{1}{\sqrt{\pi}}\cos(2t)$$
$$f_3(t) = \frac{1}{\sqrt{\pi}}\cos(3t)$$

   Verify whether $\{f_1, f_2, f_3\}$ form a complete orthonormal basis of $\mathcal{L}^{2,\text{even}}$.

For the exercises the following number of points can be obtained:

Exercise	1.	12 points	Exercise	2.	6 points	Exercise	3.	9 points

The grade is determined by adding 3 points to the total number of points obtained and dividing by 3.