UNIVERSITY OF TWENTE
Department of Electrical Engineering, Mathematics and Computer Science

Exam Signals and Transforms on Monday March 5, 2018, 8.45 – 10.15 hours.

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer!
You are not allowed to use a calculator. Besides pen and paper, the only thing you are allowed to use is one handwritten, singe-sided, A4-sized page of personal notes.

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1. Let $f(t)$ be the 4-periodic function which satisfies:

$$f(t) = e^{2(t+1)}, \qquad \text{for } t \in [-1, 3)$$

   a) Sketch the function $f(t)$ for $t \in [-5, 7]$.

   b) Show that the line spectrum of f is given by:

$$f_k = \frac{i^k(e^8 - 1)}{2(4 - k\pi i)}$$

   c) Determine the real Fourier series for $f(t)$.

   d) Is f equal to the real Fourier series for all $t \in \mathbb{R}$?

   e) Determine the generalized derivative of the function f.

2. Determine the convolution of $f(t) = e^{1-t}\,\mathbb{1}(t)$ and $g(t) = e^t\,\mathbb{1}(1-t)$ **without** the use of Fourier or Laplace transformation.

3. We consider the space $\mathcal{L}^2[0,1]$ of real valued functions on the interval $[0,1]$ for which

$$\|f\| = \sqrt{\int_0^1 f(t)^2 \mathrm{d}t} < \infty$$

   a) Consider the sequence of functions $\{g_1, g_2, \ldots\}$ in $\mathcal{L}^2[0,1]$ with

$$g_n(t) = \sin\left(\frac{t}{n}\right)$$

   Show that this sequence is convergent and determine its limit.

   b) Consider the linear mapping $\mathcal{A} : \mathcal{L}^2[0,1] \to \mathcal{L}^2[0,1]$ defined by:

$$\mathcal{A}f = h \text{ with } h(t) = t f(1-t) \text{ for } t \in [0,1].$$

   Show that $\mathcal{A}$ is a bounded linear mapping.

For the exercises the following number of points can be obtained:

Exercise	1.	12 points	Exercise	2.	6 points	Exercise	3.	9 points

The grade is determined by adding 3 points to the total number of points obtained and dividing by 3.