Signals & Transforms — TEST 2
(part of AM module 4 — 201800138)

Date:	04-06-2020
Place:	At home!
Time:	08:45–10:15 (till 10:40 for students with special rights)
Course coordinator:	G. Meinsma
Allowed aids during test:	see below

The solutions of the exercises should be clearly formulated. Moreover, in all cases you should motivate your answer! You are not allowed to use a calculator.

Integrity statement Please read the following paragraph carefully.

By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

The only allowed sources for this test are:

- the lecture notes “Signals and Transforms” (pdf or printed)
- the slides (pdf or printed)
- electronic devices, but only to be used:
  - for downloading the test and afterwards uploading your work to Canvas
  - to show the test/book/slides on your screen
  - to write the test (in case you prefer to use a tablet instead of paper to write on)

P.T.O.

A. **Copy the following text verbatim to the first page of your work (handwritten) and sign it. If you fail to do so, your test will not be graded:**

> *I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.*

B. **What programme do you follow (AM, AM+AP, AM+TCS, Minor, ...)**

C. **Are you entitled to extra time? (We will check this with CES.)**

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1. (a) Determine the Fourier transform of

$$e^{-\frac{1}{2}t^2}\sin(t).$$

   (b) Let $a \in \mathbb{R}, a \neq 0$. Determine the energy of

$$\frac{1}{a^2+t^2}.$$

2. Determine the convolution of $f(t) = e^{-|2t|}$ and $g(t) = e^{-|2t|}$ via Fourier transformation.

3. Use the Laplace transform to find the solution $y(t)$ for $t > 0$ of the integral equation

$$y(t) = t^2 + \int_0^t y(\tau)\sin(t-\tau)\,d\tau.$$

4. Given is the differential equation

$$y^{(2)}(t) - 4y(t) = u^{(1)}(t) + u(t). \qquad (1)$$

   (a) Determine the impulse response of (1).

   (b) Suppose that $u(t) = e^t$. Use Laplace transformation to determine the solution $y(t)$ for $t > 0$ of (1) for the case that $y(0^-) = 2$ and $y^{(1)}(0^-) = -2$.

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problem:	1	2	3	4
points:	4+4	6	4	4+5

Test grade is $1 + 9p/p_{\max}$

Property	Time domain	Freq. domain	Condition
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 \hat{f}_1(\omega) + a_2 \hat{f}_2(\omega)$	
Duality	$\hat{f}(t)$	$2\pi f(-\omega)$	
Conjugation	$f^*(t)$	$\hat{f}^*(-\omega)$	
Time-scaling	$f(at)$	$\frac{1}{\lvert a\rvert}\hat{f}(\frac{\omega}{a})$	$a \in \mathbb{R}, a \neq 0$
Time-shift	$f(t-\tau)$	$\hat{f}(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}$	
Frequency-shift	$f(t)\mathrm{e}^{\mathrm{i}\omega_0 t}$	$\hat{f}(\omega-\omega_0)$	
Modulation Thm.	$f(t)\cos(\omega_0 t)$	$\frac{\hat{f}(\omega-\omega_0)+\hat{f}(\omega+\omega_0)}{2}$	
Differentiation (time)	$f^{(1)}(t)$	$(\mathrm{i}\omega)\hat{f}(\omega)$	$\lim_{t\to\pm\infty} f(t) = 0$
Integration (time)	$\int_{-\infty}^{t} f(\tau)\,\mathrm{d}\tau$	$\frac{\hat{f}(\omega)}{\mathrm{i}\omega}$	$\hat{f}(0) = 0$
Differentiation (freq.)	$-\mathrm{i}tf(t)$	$\hat{f}'(\omega)$	

$f(t)$	$\hat{f}(\omega)$	Condition
$\mathrm{rect}_a(t)$	$a\,\mathrm{sinc}(a\omega/2)$	$a > 0$
$\mathrm{trian}_a(t)$	$a\,\mathrm{sinc}^2(a\omega/2)$	$a \in \mathbb{R},\ a > 0$
$\mathrm{e}^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$\mathrm{Re}(a) > 0$
$\frac{t^n}{n!}\mathrm{e}^{-at}\,\mathbb{1}(t)$	$\frac{1}{(a+\mathrm{i}\omega)^{n+1}}$	$\mathrm{Re}(a) > 0;\ n \in \mathbb{N}$
$-\frac{t^n}{n!}\mathrm{e}^{-at}\,\mathbb{1}(-t)$	$\frac{1}{(a+\mathrm{i}\omega)^{n+1}}$	$\mathrm{Re}(a) < 0;\ n \in \mathbb{N}$
$\mathrm{e}^{-(at)^2}$	$\frac{\sqrt{\pi}}{\lvert a\rvert}\mathrm{e}^{-(\omega/(2a))^2}$	$a \in \mathbb{R},\ a \neq 0$
$a\,\mathrm{sinc}(at/2)$	$2\pi\,\mathrm{rect}_a(\omega)$	$a \in \mathbb{R},\ a > 0$

$f(t)$	$\hat{f}(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\delta(t-b)$	$\mathrm{e}^{-\mathrm{i}\omega b}$
$\mathrm{e}^{\mathrm{i}\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$
$\mathrm{sgn}(t)$	$\frac{2}{\mathrm{i}\omega}$
$\mathbb{1}(t)$	$\frac{1}{\mathrm{i}\omega} + \pi\delta(\omega)$

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Time-scaling	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$ (if $a > 0$)
Time-shift	$f(t-t_0)\,\mathbb{1}(t-t_0^-)$	$F(s)\mathrm{e}^{-st_0}$ (if $t_0 > 0$)
Shift in s-domain	$f(t)\mathrm{e}^{s_0 t}$	$F(s-s_0)$
Differentiation (t)	$f^{(1)}(t)$	$sF(s) - f(0^-)$
	$f^{(2)}(t)$	$s^2F(s) - sf(0^-) - f^{(1)}(0^-)$
Integration (t)	$\int_{0^-}^{t} f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
Differentiation (s)	$-tf(t)$	$F'(s)$

$f(t),\ (t > 0^-)$	$F(s)$
e^{at}	$\frac{1}{s-a}$
$\frac{t^n}{n!}$ $(n \in \mathbb{N})$	$\frac{1}{s^{n+1}}$
$\frac{t^n}{n!}\mathrm{e}^{at}$ $(n \in \mathbb{N})$	$\frac{1}{(s-a)^{n+1}}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$\mathrm{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\mathrm{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$\delta(t)$	1