Robust Control — EXAM

Course code:	191560671
Date:	18-04-2017
Time:	13:45–16:45 (till 17:30 for students with special rights)
Course coordinator & instructor:	G. Meinsma
Type of test:	open book
Allowed aids during the test:	printed lecture notes, basic calculator

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1. Consider the non-rational transfer function

$$G(s) = \frac{1}{1+\frac{1}{2}\mathrm{e}^{-s}}$$

defined for those $s \in \mathbb{C}$ for which $1+\frac{1}{2}\mathrm{e}^{-s} \neq 0$.

   (a) Show that $G \in \mathbb{H}_\infty$.

   (b) Determine $\|G\|_{\mathbb{H}_\infty}$.

2. Not infrequently disturbances w enter the plant at the input (see w in the second figure on page 25 of the notes). Suppose that the system is internally stable and that the loopgain has integrating action (i.e. $L(s)$ has a pole at $s=0$). Prove that the DC-gain of $H_{y/w}(s)$ is zero if and only if the *controller* has integrating action.

3. Chapter 4 introduces the *gain margin* k_{m}, *phase margin* ϕ_{m} and *modulus margin* s_{m}.

   (a) Show that $0 < s_{\mathrm{m}} < 1$ implies a guaranteed gain margin of at least $1/(1-s_{\mathrm{m}})$.

   (b) Does $k_{\mathrm{m}} < 1$ imply a guaranteed $s_{\mathrm{m}} > 0$?

   (c) Does $\phi_{\mathrm{m}} > 0$ imply a guaranteed $s_{\mathrm{m}} > 0$?

4. In § 8.1 we designed a stabilizing controller for the plant $P(s) = 1/s^2$. Unfortunately all sensitivity functions S, T designed in § 8.1 appear to have peaks with $\|S\|_{\mathbb{H}_\infty} > 1$ and $\|T\|_{\mathbb{H}_\infty} > 1$.

   (a) For this plant is there a stabilizing $K(s)$ that achieves $S(0) = 0$ and $\|S\|_{\mathbb{H}_\infty} \leq 1$?

   (b) For this plant is there a stabilizing $K(s)$ that makes $T(s)$ strictly proper and achieves $\|T\|_{\mathbb{H}_\infty} \leq 1$?

   Explain your answers.

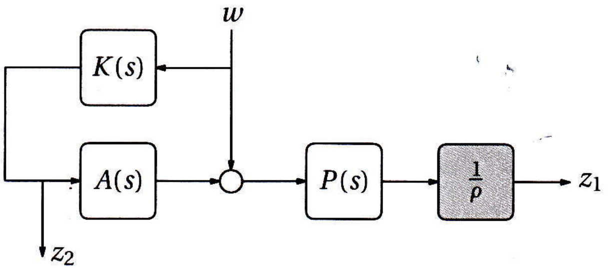
5. *Chapter 6:* Consider the system

$$\dot{x} = \frac{1}{4}x + u, \qquad x(0) = x_0.$$

with cost $\int_0^\infty x^2(t) + (x(t) + u(t))^2 \, dt$. Determine the solution P of the corresponding LQ-Riccati equation and determine the LQ-optimal state feedback $u = -Fx$.

6. Are all polynomials in the family of polynomials $[1,2]s^3 + [1,2]s^2 + [1,2]s + [1,2]$ stable?

7. *Disturbance feedforward.* Sometimes we can measure a disturbance w that acts on a plant $P(s)$. It then makes sense to try to counter-act this disturbance. Hagander and Bernhardsson suggested the following scheme:



in which $\rho > 0$ is a tuning parameter and $A(s)$ is some given actuator system, and then they suggest to minimize $\|H_{z/w}\|_{\mathbb{H}_\infty}$ over all stabilizing $K(s)$.

(a) Under what conditions on $A(s), P(s), K(s)$ is this system internally stable?

(b) Formulate this ~~is~~ as a standard $\mathbb{H}_\infty$ problem (that is, determine the generalized plant $G(s)$).

(c) Suppose that $P(s) = A(s) = 1/(s+1)$. What can you say about the order of the $\mathbb{H}_\infty$-optimal controller K? (For proper rational $K(s)$ the "order" is the degree of its denominator polynomial.)

8. Table 9.1 of the lecture notes claims that the interconnection matrix for $P = P_0(I + V\Delta W)^{-1}$ is $H = -W(I + KP_0)^{-1}V$. Verify this result. Your derivation must hold for MIMO systems as well.

problem:	1	2	3	4	5	6	7	8
points:	2+2	3	2+2+2	2+3	4	3	2+2+3	4

Grade: $= 1 + 9\frac{p}{p_{\max}}$ (possibly with homework correction of ≤ 0.6)